Magicians have invented a fantastic variety of self-working mathematical tricks in which the outcome of seemingly random choices can be predicted in advance. Some of these tricks are described in my 1956 Dover paperback *Mathematics, Magic, and Mystery*. However, hundreds of new mathematical tricks have been invented since that book was published. What follows is a small selection.

Readers are urged to follow the instructions of each trick carefully, then turn to page 26 for a prediction of the outcome. I will not spoil the fun by explaining why each magic trick works. If you can figure this out for yourself, you'll find the tests pleasant exercises in mathematical problem solving.

1. **Twinkle Twinkle**

   Twinkle, twinkle, little star,
   How I wonder what you are
   Up above the world so high,
   Like a diamond in the sky.

   Twinkle, twinkle, little star,
   How I wonder what you are.

   **Select any word in the first two lines of the above familiar poem. Count the number of its letters. Call this number \( n \). Now count ahead \( n \) words, starting with 1 on the word following the word you selected. Count the number of letters in this second word. Call the number \( k \). Count ahead \( k \) words to arrive at a third word.

   Continue in this manner until you can't go any farther.

   On what word does your count end?**

2. **246,913,578**

   Enter the above strange number in your calculator. You may freely choose to do any of the following:

   Multiply the number by 2, 4, 5, 7, 8, 10, 11, 13, 16, 20, 22, 25, 26, 31, 35, 40, 55, 65, 125, 175, or 875.

   Or, if you prefer, divide the number by 2, 4, 5, or 8.

   After you have done one of the above, rearrange the digits of the result in serial order from the smallest digit to the largest. Ignore any zeros among the digits.

   The result will be a number of nine digits.

   What is this number?

3. **Try This On a Dollar Bill**

   Write down the serial number of any dollar bill. Scramble the digits any way you like—that is, mix up their order. Jot down this second number.

   Using your calculator, subtract the smaller number from the larger.

   From the difference, subtract 7.

   Copy the digits now on display, then add them all together. If the sum is more than one digit, add the digits once more.

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**Martin Gardner** (aka "Mr. Mathematical Games") has written more than 60 books on mathematics, magic, Alice in Wonderland, and scores of other topics.
Keep adding digits in the sum until just one digit is obtained.
What is it?

4. Count the Matches

From an unused folder of 20 paper matches, tear out any number of matches fewer than ten and put them in your pocket.
Count the number of matches remaining in the folder. Add the two digits of the count, then remove that number of matches from the folder. Put them in your pocket.
Tear out three more matches.
How many matches are left in the folder?

5. A Test With Two Dice

Roll a pair of dice on the table. Call them A and B.
Write down the following four different products:
1. The product of the top numbers on the dice.
2. The product of their bottom numbers.
3. The product of the top of A and the bottom of B.
4. The product of the top of B and the bottom of A.
Add the four products.
What’s the sum?

6. Fold and Trim

Fold a sheet of paper in half four times, and then unfold it. The creases will form a 4-by-4 matrix of cells as shown in Figure 1. Number the cells from 1 through 16 as illustrated. Fold each crease forward and back a few times so the paper will fold easily either way along each crease.

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Now fold the sheet into a packet the size of one cell. You can make the folds as tricky as you please, folding any way you like. You may even tuck folds between folds. In other words, make the folding as random as possible until you have a packet the size of a single cell.
With scissors trim away all four edges of the packet so that it consists of sixteen separate pieces. Spread these pieces on the table. Some will have their numbers side up, others will have their numbers side down.
Add all the numbers on the face-up pieces.
What is the sum?

7. At the Apex

Copy the triangle of circles in Figure 2. Put any four digits you like in the four vacant circles of the bottom row. They needn’t be all different, and you may include one or more zeros if you like.

8. The Red and the Black

Shuffle a deck of cards, then deal 30 cards to the table to form a pile.
Count the number of black cards in the pile. From this number subtract the number of red cards in the rest of the deck.
What’s the difference?
9. Four File Cards

Write 39 on one side of a file card, and 51 on the other side. On a second file card write 26 and 34 on the two sides. A third card gets 65 and 85. A fourth card gets 52 and 68.

Place the four cards on the table so the numbers showing on top are 26, 39, 52, and 65.

Slide any card out of the row, then turn over the three remaining cards. Slide out another card, and turn over the remaining two cards.

Slide out a third card. Turn over the card that remains.

You now have a choice of leaving the cards as they are, or turning all of them over.

With your calculator, multiply all the numbers showing. What is the product?

10. A 3 by 4 Test

Copy the 3 by 4 matrix in Figure 3.

Put digits 1 through 9 in the cells in any way you like. Three empty cells will remain. In those cells put either three ones, or three fours, or three sevens.

Treat each row of the matrix as a 3-digit number. Add these four numbers by writing the sum over the four lines below the matrix.

Add the sum's four digits. If the result is more than one digit, add those two numbers. Keep adding until only one digit remains.

What is this digit? ■

From the MAA

New Mathematical Diversions
Martin Gardner

Required reading for students, teachers, mathematicians at all levels, as well as interested laypersons.

This lighthearted treatment of serious mathematics can form the basis of some very useful material for mathematical teachers ... Because the 20 chapters were previously published, Gardner includes a postscript section at the back of the book to update any material from its original publication. This section shows that elementary mathematics is alive and that when presented in recreational form, continues to excite a large readership ... Every mathematics teacher should read and "work" this book thoroughly!

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Answers are provided for problems, as well as references for further reading and a bibliography. The Postscript section provides updates to the problems.

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Back to the Future

Although I’m not using the specific collection of facts from logic that I accumulated while a graduate student and assistant professor, many of the skills I gained then serve me well now. One important example is the ability to pick up new mathematical ideas quickly—I’m learning a great deal about statistics, mathematical modeling, time series analysis, and modern regression techniques. But perhaps most critical is the ability to think creatively about new mathematical relationships.

One constant of the financial world is that individual investment strategies become less effective over time. For example, it used to be the case that bond prices would go down just before the U.S. government auctioned off a new batch of treasury bonds and then go back up afterwards. But as more and more people tried to buy bonds before the auction and sell them afterwards to profit from this pattern, bond prices were gradually driven up before the auction and driven down afterwards, until the effect disappeared altogether. Therefore, to be successful, a company like ours must always seek new, more subtle relationships among the world’s investments. I’m confident that however much I learn about the way markets work, there will always be fresh intellectual challenges for me in mathematical finance.


Statistician (continued from p. 24)

degree. A strong mathematics background will help ensure that a student earns the master’s degree in a timely fashion. In a graduate program you study statistical techniques in greater detail and get exposure to many of the sub-disciplines or sub-areas of statistics such as sampling, nonparametric, and multivariate statistics. This enables you to use statistics to solve more complex problems in a wider array of areas, and thereby broadens your career opportunities. Many universities also offer a doctorate in statistics, which is essential for those interested in teaching and research at the college or university level.

Though the master’s degree in statistics is the appropriate degree for most people who plan on a business or industrial career, the doctorate in statistics will enhance one’s opportunity for advancement. A doctorate in statistics continues where the master’s degree leaves off by exposing you to statistics in much greater depth and in more areas.

With a graduate degree in statistics a person is well prepared for employment as a statistician in many different areas. Basically there are three main areas that hire statisticians: business or industry, government, and academia, with business and government hiring the majority of the statisticians. On the business side, there are certain industries that support the greatest concentration of statisticians. Pharmaceutical companies and medical research organizations use statisticians to assist in developing new treatments or drugs for human disorders. Public opinion businesses (such as the Gallup and Harris Polls and the Nielsen rating service) use statisticians to collect and analyze data. Any production company will employ statisticians as quality control technicians or engineers to monitor and improve their production processes. Government statisticians are involved with surveys such as the census, employment studies and agricultural studies, and, as one might expect, are employed in agencies such as the Census Bureau, the Bureau of Labor Statistics, the Environmental Protection Agency, the Agriculture Depart-

ment, and the National Institute for Occupational Safety and Health. Finally, statisticians are also hired in an academic setting in non-teaching positions. Many large universities have research units where statisticians are involved in a variety of capacities. For example, most medical schools have consulting centers, comprised of statisticians that help in the design and analysis of data from medical research projects.

As you have now discovered, employment opportunities for statisticians are plentiful and are available in all geographic areas of the country. Life as a statistician can be very rewarding as you work with people in other fields to solve important problems. It is one of the important fields that a mathematics major can consider for a career. Additional information on careers in statistics is available from the web site of the American Statistical Association (http:/www.amstat.org).

This article on advising mathematics majors about opportunities for careers in statistics is part of a series of articles sponsored by the MAA Committee on Advising. The authors appreciate the helpful comments received from members of this committee.

Solution to Chess Problem

(From Noam Elkies on p. 10.)

1. h4! e2 (Ke2? 2 B:d1 + Ke1 3 Bf3 + d1 = anything 4:d1#)
2. Be2! d:c2
3. Rb3 c1 = anything
4. Rh3 any
5. Rh1 mate!

Predictions

(From Gardner’s Gatherings on p. 13–15.)

1. The count ends on the word “you.”
   The trick is based on what magicians call the Kruskal Count, a principle discovered by Princeton mathematician Martin Kruskal.
2. The number is 123456789.
3. The final digit is 2.
4. Six matches remain in the folder.
5. The sum of the four products is 49.
6. The face-up numbers add to 68.
7. The triangle’s top number is 4.
8. The difference between the black and red cards is 4.
9. The product of the four numbers is 5,860,920.
10. The final digit is 3.