

A Magic Trick Based on the Hamming Code

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In 2006, Richard Ehrenborg introduced a magic trick based on properties of the Hamming code. Here we improve upon the trick, allowing the magician to predict both a number and color selected by the volunteer, and to perform the trick more easily in a public setting.

The magician walked into the spotlight and faced the audience. He asked for a volunteer. The young lady who raised her hand first was asked to pick a number from 1 through 15 and a color from the set {red, orange, yellow, green, blue, pink, purple}. She was then shown seven colored cards and asked to report wheth-

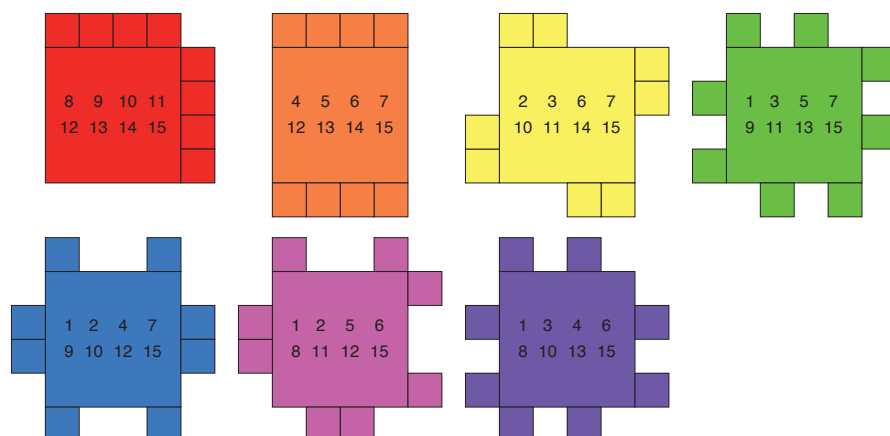


Figure 1. Cards used for the Hamming code magic trick.

er or not her chosen number appeared on each card. On the card of her chosen color, however, she was instructed to lie. After giving her responses, the magician reported both her chosen number and her chosen color. The audience applauded and wondered how the magician was able to make these determinations.

As it turns out, there is nothing magic about this trick at all. It is based on the mathematics of error-correcting codes. The same principles explain how CDs, DVDs, and Blu-ray Discs can still play when scratched.

HOW THE TRICK WORKS

The seven cards used by the magician are shown in figure 1.

	0	1	2	3	
15					4
14					5
13					6
12					7
	11	10	9	8	

Figure 2. Base card for the Hamming code magic trick.

The seven cards from figure 1 are placed on a base card as shown in figure 2.

A magician learning this trick can use a base card with the 15 numbers displayed. Eventually a base card without the numbers can be used so that it is harder for the audience to figure out what is going on. The magician simply needs to memorize the locations of the numbers on the base card. This should not be difficult since the numbers appear in clockwise order starting in the upper left corner.

Whenever the volunteer gives a YES response for a card, it is placed on top of the base card. For example, if the chosen number was 14, figure 3 shows the result of placing the red card on the base

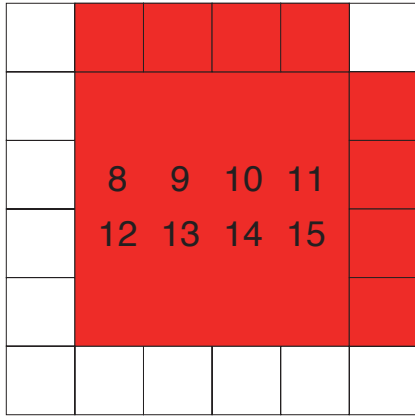


Figure 3. Result of YES response with chosen number 14.

card. By comparing figures 2 and 3, we see that numbers 0–7 have been covered up in red while positions 8–15 still reveal the white base. The numbers written on each colored card are precisely the numbers on the base that are left uncovered by that card.

Whenever the volunteer gives a NO response for a card, that card is flipped across a diagonal from the upper left to the lower right, as shown in figure 4, before being placed atop the base card.

Observe that all the numbers that would be covered by the card in its original orientation will be revealed in the flipped state, and vice versa.

If the volunteer truthfully responds to the first four cards, then figure 5 shows the result of the responses to these cards with the selected number 14. (Here the

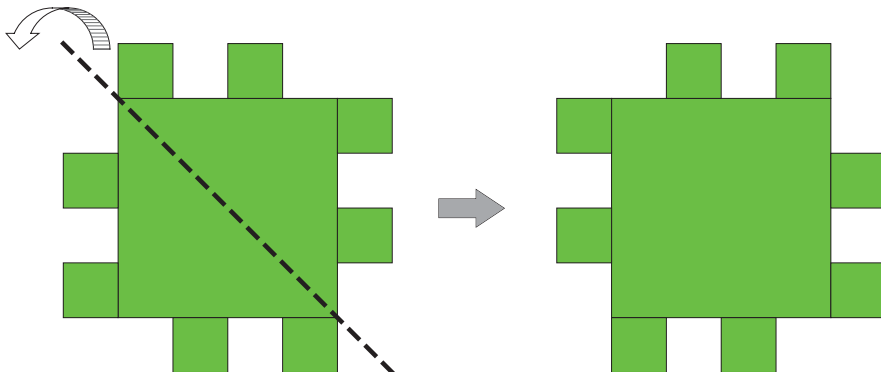


Figure 4. Result of flipping one of the colored cards.

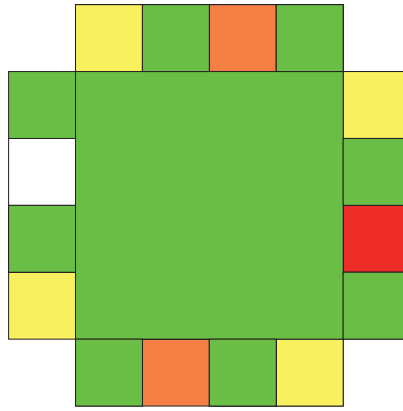


Figure 5. Result of four truthful responses with selected number 14.

responses would be: yes, yes, yes, no.)

Observe that only position 14 is exposed on the original white base card while one or more of the first four cards have covered each of the other positions. Assuming that all four responses were truthful, these four cards implement the binary representation of the selected number and can be used as a magic trick introducing binary arithmetic.

If all seven cards are used and one incorrect response is given, the position of the chosen number will be covered by exactly one card, while the remaining 15 positions will be covered by at least two cards. In his article “Decoding the Hamming Code” (*Math Horizons*, **13** [4], 2006, 16-17), Ehrenborg claims that finding the position covered by one card is straightforward. However, in practice it is

not so simple; there is no way to determine this location without handling the cards, which makes the trick less impressive.

Note that each card has eight tabs—those tiny squares that stick out from the sides.

To implement the color identification *and* overcome the difficulty in determining the region covered by a single card, we punch a tiny hole in every tab on every card. Figure 6 illustrates a scheme for placing a hole on a single tab.

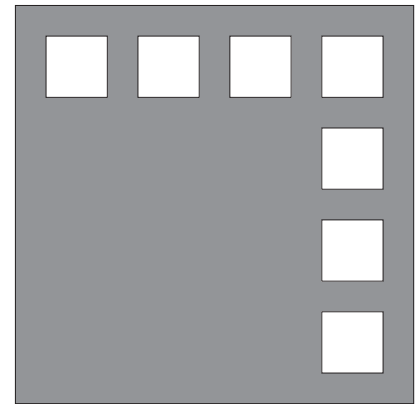


Figure 6. Possible locations of the tab holes on a single tab.

Each card must have its own unique tab-hole position—one of the seven positions shown in figure 6—and every tab on that card must have a hole placed in this same relative position. The important point is that when stacked, no two cards share a tab-hole position, even if one of the cards is flipped. After implementing this scheme, the magician will observe a single white hole at the conclusion of the trick. The location of the white hole corresponds to the volunteer’s selected number, and the color of the tab surrounding the hole gives the volunteer’s selected color.

For example, suppose that the number 14 was selected along with the color pink. At the conclusion of the trick, the magician observes the situation shown in figure 7.

Here, we see one white hole in

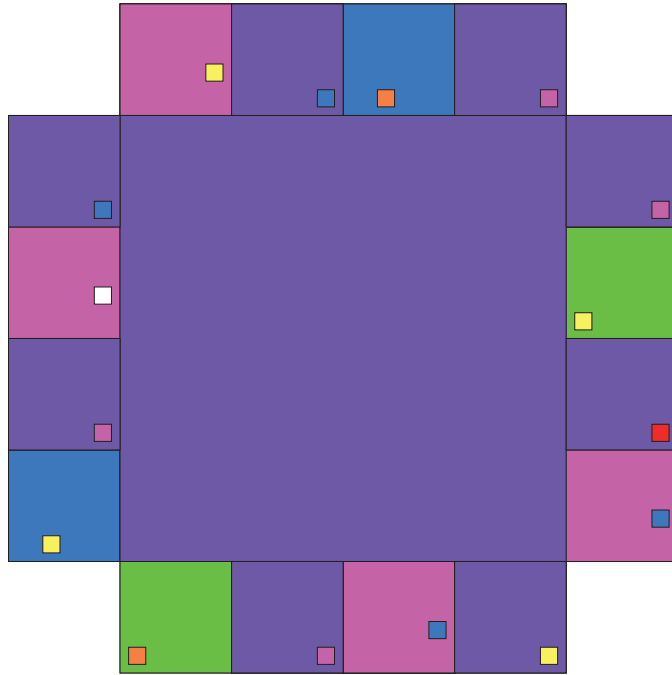


Figure 7. Stack of all seven cards with selected number 14 and color pink.

position 14 and the color pink surrounding the hole. So the magician identifies this number and color as the volunteer's selection.

WHY THE TRICK WORKS

This trick is based on the (7,4) Hamming code with the following generator matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

The (7,4) notation means that a total of 7 bits (yes/no responses) are needed to encode a total of 4 bits. The numbers 0–15 can be represented in binary using these four bits. Each column of this matrix corresponds to one of the seven cards. By finding all linear combinations

of the rows of this matrix, we obtain the 16 code words given in figure 8:

- 0 : [0 0 0 0 0 0 0] (0)
- 1 : [0 0 0 1 1 1 1] (4)
- 2 : [0 0 1 0 1 1 0] (3)
- 3 : [0 0 1 1 0 0 1] (3)
- 4 : [0 1 0 0 1 0 1] (3)
- 5 : [0 1 0 1 0 1 0] (3)
- 6 : [0 1 1 0 0 1 1] (4)
- 7 : [0 1 1 1 1 0 0] (4)
- 8 : [1 0 0 0 0 1 1] (3)
- 9 : [1 0 0 1 1 0 0] (3)
- 10 : [1 0 1 0 1 0 1] (4)
- 11 : [1 0 1 1 0 1 0] (4)
- 12 : [1 1 0 0 1 1 0] (4)
- 13 : [1 1 0 1 0 0 1] (4)
- 14 : [1 1 1 0 0 0 0] (3)
- 15 : [1 1 1 1 1 1 1] (7)

Figure 8: The 16 code words of the (7,4) Hamming code.

To the left of each code word is a number specifying the base 10 equivalent of the binary representation of its first four components. This number appears on

each of the cards where the code word has nonzero entries.

To the right of each code word is the total number of ones in the code word. Observe that the smallest count for any nonzero code word is three. Because this is a so-called *linear code*, this also means that any two of the code words in figure 8 must differ in at least three components.

Since there are at least three differences between any two code words, it is possible to correct any error in a single component of any code word. One method of decoding a received vector is to compare it with each of the 16 code words and select the closest one—the code word that differs from the received vector in the fewest places. The single white hole at the end of the magic trick allows the magician to make the 16 comparisons quickly without moving the cards.

A computer simulation of this trick as well as templates for making the cards are available at maa.org/mathhorizons/supplemental.htm. ■

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FURTHER READING

R. W. Hamming, "Error detecting and error correcting codes," *Bell System Tech. J.* **29** (1950) 147–160.