BLACKSTONE'S MATHMAGIC

On an episode of the PBS series "Square One," Blackstone the magician performed the following "trick."

He first wrote down a number on a folded piece of paper. He concealed it from view, and then displayed a poster containing the numbers shown in **figure 1a.** A person was asked to pick a number from row 1, say 3, and then to draw a vertical line crossing out the remaining numbers in this column **(fig. 1b).** The person then was asked to repeat the procedure for row 2, and so on **(fig. 1c)**, and then to add the chosen numbers. Of course the sum of 34 was shown to be the number that the clairvoyant Blackstone had written on the paper.

I asked a first-year-algebra class, "Does '34' always come out as the sum?" and "Can we generalize this idea to find a sum for square arrays of other dimensions?"

Students made other number selections to confirm the sum. After trying these selections, the students noted that the four numbers chosen must not only come from four different rows but also from four different columns. At this point, we decided to rename the entries in the square array in terms of multiples of 4 to help identify patterns, as seen in **figure 1d.** Since every choice of four numbers must contain a number from each row and each column, we realized that the sum must be of the form

$$4(1) + 4(2) + 4(3) + 4(4) - 3 - 2 - 1.$$

We looked at the five-by-five array containing the numbers from 1 to 25. We then modified the directions: pick *any* number, then cross out the other numbers in the same row and the other number in the same column as your chosen number. (See **fig. 1e.**) Each student chose her or his own set of five numbers, and as before, one number from each row and each column was obtained. At this point, just about every student was able to discover that not only was the sum 65, but in this five-by-five case the sum was always

$$5(1) + 5(2) + 5(3) + 5(4) + 5(5) - 4 - 3 - 2 - 1.$$

I asked for alternative ways of computing this sum, and one of the students, recalling the distributive property, gave us the grouping

$$5(1 + 2 + 3 + 4 + 5) - (1 + 2 + 3 + 4).$$

At this point, I asked them to work with a partner and find a way to add the first four, five, or six counting numbers and reach a generalization. Within five minutes, at least four pairs of students had come up with the proper procedure, which I prodded them to formalize into an apparent theorem: *The sum of the first n natural numbers is given by*

$$\frac{n(n+1)}{2}.$$

We were then able to demonstrate that the five-by-five square gives a sum of

$$5(1 + 2 + 3 + 4 + 5) - (1 + 2 + 3 + 4)$$
$$= 5\left(\frac{5(6)}{2}\right) - \frac{4(5)}{2} = 75 - 10 = 65.$$

The students were eager to try higher numbers. They worked in pairs again on various squares and found that a six-by-six array gave a sum of 111 and that a ten-by-ten array gave a sum of 505.

The class made a fairly straightforward jump to the n-by-n array yielding the sum

$$n(1 + 2 + 3 + 4 + \cdots + n) - (1 + 2 + 3 + \cdots + (n - 1))$$

But we had previously found a shortcut to adding the sum of the first n natural numbers. Our n-by-n sum could then be written

$$n\left(\frac{n(n+1)}{2}\right) - \frac{(n-1)(n)}{2}.$$

They were asked to "simplify the expression" and check their answer with their partner. The students then verified the formal result with the

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"Sharing Teaching Ideas" offers practical tips on teaching topics related to the secondary school mathematics curriculum. We hope to include classroom-tested approaches that offer new slants on familiar subjects for the beginning and the experienced teacher. Of particular interest are alternative forms of classroom assessment. See the masthead page for details on submitting manuscripts for review.

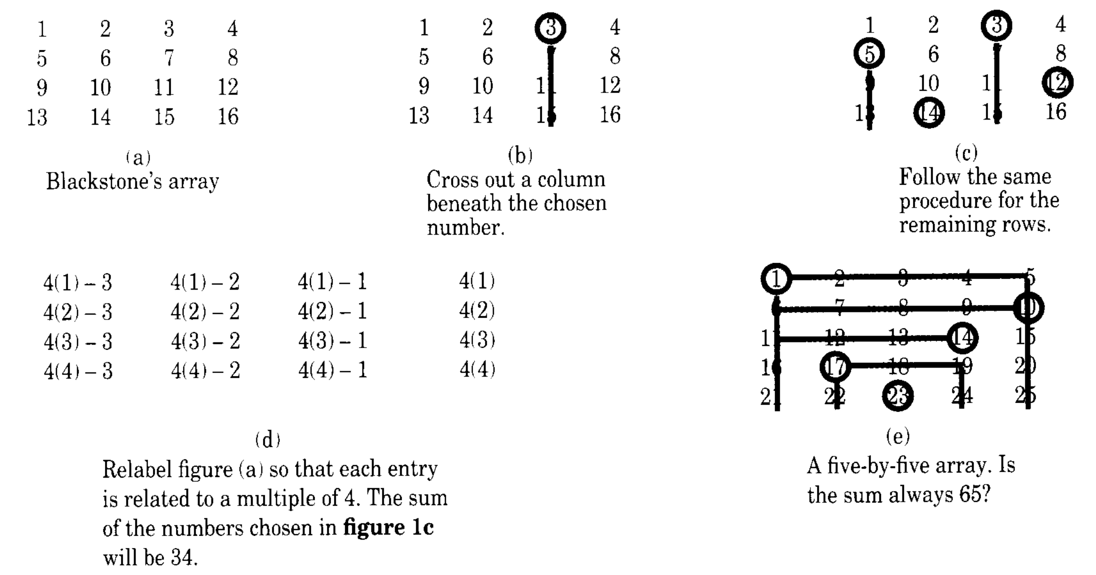

Fig. 1

experimental results from the numerical examples. Indeed

$$\frac{(n^3 + n)}{2}$$

supplies the correct sum regardless of the dimension.

I especially liked this problem because neither I nor the students knew the "answer" beforehand. Also, each step was fairly easy for them to comprehend. We all enjoyed the thrill of discovery.

James W. Roche
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GRAPHING A SOLID: A CLASSROOM ACTIVITY

Three-dimensional graphing can present a simple and interesting approach to studying geometric solids. Three-dimensional coordinates and a distance formula are used to generate models that students can build. Here is a ready-to-use lesson that puts a complex model on the desk of each student.

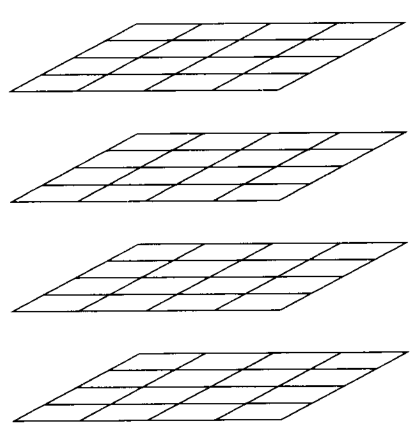

Fig. 1
A graphic representation of four parallel planes

Give each student a reproduction of **figure 1,** a graphic representation of four parallel planes. The students can become familiar with the perspective of this figure by playing three-dimensional tic-tac-toe on it. This game is simple. Two or three students compete against each other using **figure 1** as the playing field. In turn, each student places a symbol of his or her choice in one of the squares. The symbol can be an X, an O, a $, the student's initial, or a smiley face. The first player to complete four collinear symbols wins.

Figure 2 shows a completed three-player game that the student with the $ symbol has won. This activity should stimulate discussions of collinearity and the perspective of the figure. I keep a plastic model of this game in front of the class for reference.

When students are familiar with this representation of four parallel planes, they add a few extra lines and some numbers to convert the tic-tac-toe