ALGEBRA AND A SUPER CARD TRICK

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Here is a fascinating card trick that can be explained and justified using first-year algebra. We have used this trick with high school classes and mathematics clubs as a motivational device and sometimes as a challenge to students to find or finish the algebraic justification. This procedure makes a good example of the power of mathematics to unmask seemingly complex situations and, therefore, is a good device for teachers to "keep up their sleeves" for some auspicious occasion or a time when interest is lagging. Some related procedures can be found in the bibliography.

Get a standard fifty-two-card deck and work through each step. Later we shall examine the algebra behind the scenes. We assume that jacks, queens, and kings have values of 11, 12, and 13, respectively, whereas aces have a value of 1.

1. Shuffle the deck and begin placing cards faceup all in one stack on a desk top. Claim that you are memorizing the sequence of cards displayed. Challenge the spectators to perform this great mental feat as well!

2. As you are placing cards faceup in step one (say after you have dealt about a dozen cards), secretly pick out and remember one card. Then continue the dealing process so that you place exactly *ten* more cards faceup on top of your secretly selected card (see fig. 1).

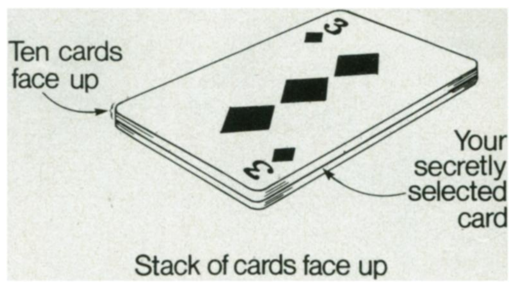


Fig. 1

3. Have each of three students select one card at random from the cards in your hand. Have these three cards placed faceup in three separate locations on the desk top. Let's assume the students selected a 4, an 8, and a jack.

4. Turn over the face-up pile of cards containing your secretly selected (and memorized) card and place it under the stack of cards in your hand. You now have three cards, each facing up, and one stack of cards in your hand.

5. Now work with each face-up card separately. Place additional cards facedown on top of each of the three face-up cards. Start with the value of each face-up card and add cards until you reach a count of thirteen. Let the bottom face-up card stick out a little so you can use it in the next step (see fig. 2).

Face-down cards counted as shown

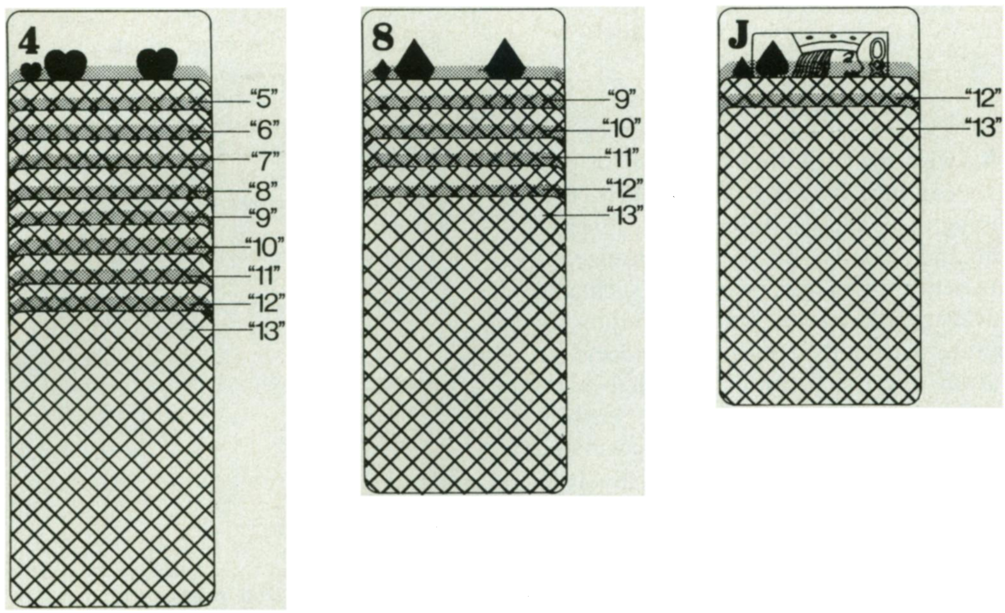


Fig. 2

6. Now, out loud, add the values of the three face-up cards on the bottom of the piles in view. We shall call this sum S. (In our example we have $S = 4 + 8 + 11 = 23$.)

7. Ask if anyone knows the value of the Sth card in your hand. Pretend you are struggling to recall it—remember, you claimed to have memorized a long sequence of cards—and announce it as if you were able to remember it! Count out S cards, and you can't miss!

A Rationale

Let's begin by finding an expression for the total number of cards in each of the three piles of cards left in view. We had piles on top of 8, 4, and a jack.

On the 8 pile, we had $6 = 14 - 8$ cards.

On the 4 pile, we had a total of $10 = 14 - 4$ cards.

On the jack pile, we had a total of $3 = 14 - 11$ cards.

In general, if n is the value of the face-up card on the bottom, there are $14 - n$ cards in the pile. If n_1, n_2, and n_3 are the values of the face-up cards on the bottom, then there are

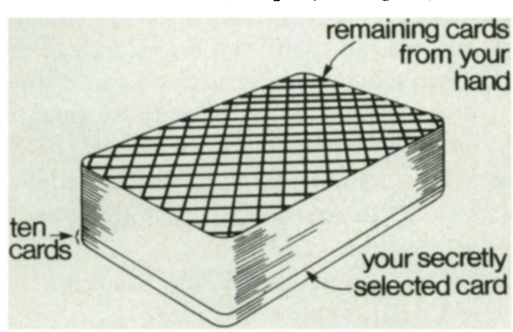
$$(14 - n_1) + (14 - n_2) + (14 - n_3)$$

cards on the table. In step 7 you counted $S = n_1 + n_2 + n_3$ cards down into the cards in your hand. Adding the number of cards on the table and S gives

$$(14 - n_1) + (14 - n_2) + (14 - n_3) + (n_1 + n_2 + n_3) = 42.$$

You will always reach card number forty-two.

How many cards are beneath your secretly selected card? Ten—you put them there in the second step! (See fig. 3.) That



Stack of cards face down

Fig. 3

means your secretly selected and memorized card is also card number forty-two.

Teacher's Corner

As with all card tricks, a dash of theatrics can amplify the positive effects. Pretending to struggle to recall the memorized card and claiming to have memorized a long sequence of cards are two such maneuvers. Holding off an explanation or limiting the number of "performances" on a given day are other devices that could spark additional interest. If the class is challenged to find a rationale, they will find it easier if they know all the steps involved. Once students know how to do the trick, they are usually very interested in finding out why it works.

Here are some questions and suggestions we have used to help students find the algebraic explanation.

- Look at the total number of cards in each of the three piles. How can you predict each total if you know the value of the bottom face-up card?
- Think of the cards when they are in the three piles as all being in one stack—this includes the cards in your hand. How far down in the deck is your secret card?

Here are some questions we have posed to get students to look back at their rationales.

- Will the trick always work, or can someone pick three cards at random that will cause it to fail?
- Suppose we count to fourteen instead of thirteen as we place cards on top of the three selected cards. What other change do we have to make?
- Why is the number of cards in each pile $14 - n$? If we count to 13, it seems as if we should have $13 - n$.
- What could happen if four students selected cards in the third step?
- What changes could you make in the trick if you had a double deck (104 cards) to work with?

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