Positive Primitive Torion

In [MR], Martsinkovsky and Russell introduced a new concept of, what they call injective torsion and cotorsion.

Given an arbitrary ring $R$, let $E: R\rightarrow E$ be an injective envelope. Then $\Phi : R\text{-Mod} \rightarrow R\text{-Mod}$ is the subfunctor of $1$ that is the kernel of $E \otimes 1$ and $\nu^{-1}$ is the quotient functor of $1$ given as $1/\nu^{-1}$, where $\nu^{-1}$ is the trace of the injective left $R$-modules.

Call a unary APP formula (or the corresponding functor from $R\text{-Mod} \rightarrow Ab$) low if it vanishes on all flats and high if it covanishes on all injectives (i.e., restricted to injectives the pp functor is $1$).

Thus $\Phi$ is the sum of all low pp functors.

Let $\mathcal{N}$ be the intersection of all high pp functors. Also $\mathcal{N}$ is a functor from $R\text{-Mod} \rightarrow R\text{-Mod}$, called the Ulm functor, as it yields the first Ulm subgroup if $R = \mathbb{Z}$. Iterate $\mathcal{N}$ (as in $Ab = \mathbb{Z}\text{-Mod}$) and denote the intersection of all iterates by $\mathcal{N}^{\infty}$.

Thus $2$. $\nu^{-1} \leq \mathcal{N}^{\infty}$

Thus $3$. Restricted to pure injective modules, we have $\nu^{-1} = \mathcal{N} = \mathcal{N}^{\infty}$ and thus (known for $Ab$) $\nu = 1/\mathcal{N}$ (on pure-injectives!).

This joint work in progress with Alex Martsinkovsk.


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