On Finite Dimensional Division Algebras

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Abstract. A non-zero vector space A over a field k, endowed with a k-bilinear multiplicative structure $A \times A \to A,\ (x, y) \mapsto xy$, is called a *division algebra* in case the linear operators $L_a : A \to A,\ L_a(x) = ax$ and $R_a : A \to A,\ R_a(x) = xa$ are bijective for all $a \in A \setminus \{0\}$. We introduce to the theory of finite dimensional division algebras by firstly presenting some generalities on the category $\mathscr{D}(k)$ of all finite dimensional division algebras over an arbitrary ground field k, secondly discussing the question in which dimension an object $A \in \mathscr{D}(k)$ exists, and thirdly taking a closer look at $\mathscr{D}(\mathbb{R})$, winding up at the recently discovered double sign decomposition of $\mathscr{D}_2(\mathbb{R}), \mathscr{D}_4(\mathbb{R})$ and $\mathscr{D}_8(\mathbb{R})$ into four blocks each.

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