Irreducible Morphisms
by Raymundo Bautista

Irreducible morphisms were first studied by Maurice Auslander and Idun Reiten in connection with the almost split sequences in the category of finitely generated left (or right) modules over an Artin Algebra. Irreducible morphisms can be considered in any category, although they do no always exist. During the talk some simple examples of existence and non existence will be presented. The irreducible morphisms are related to almost split sequences in the category of finitely generated left modules over some Artin Algebra in particular for path algebras over oriented graphs.

When almost split sequences exist the properties of the irreducible morphisms are encoded in some oriented graph, called the Auslander-Reiten quiver.

As an illustration of the interrelation of the geometric properties of the Auslander-Reiten quiver and the properties of a root system of a semisimple Lie Algebra we consider the following. Take Γ some given Dynkin diagram, A the path algebra of this diagram with some orientation and $\mathcal{AR}(\mathcal{A})$, the Auslander-Reiten quiver of the category of the finitely generated A-modules. We will give a relation between the Coxeter number of Γ and the distance between the projective cover of a simple module and the injective envelope of the same simple in $\mathcal{AR}(\mathcal{A})$.

Tame and Generically Tame Algebras

If A is a finite-dimensional algebra over an arbitrary field k, a generic module is an indecomposable left A-module whose length over its endomorphism ring (its endolength) is finite, but its dimension over k is infinite. Generic modules play a central role in the description of the indecomposable left A-modules, when A is a Tame Algebra over an algebraically closed field.

This talk is a survey of results on Tame Algebras, we first consider finite-dimensional Tame Hereditary Algebras with two simples over arbitrary fields and the parametrization of the regular finite-dimensional modules using unique factorization domains (in general non-commutative), results obtained by C. Ringel and W. Crawley-Boevey. We will see the relation between the parametrization of the regular modules and the generic modules. Then we recall for Tame Algebras over algebraically closed fields, the relation between generic modules, families of one-parameter families of indecomposable modules and homogeneous indecomposable modules. Finally we consider algebras over k, an infinite perfect field, which are generically tame, this is there is only a finite number of generic indecomposable modules for each endolength. We will see that in this case (joint work with E. Pérez and L.Salmerón) the description of the indecomposable modules of any given dimension can be given in terms of a parametrization using finite-dimensional Tame Hereditary Algebras with two simples and tensor algebras $T_D(V) = D \oplus V \oplus V \otimes V \oplus \cdots$ where D is a division ring containing k in its center and finite-dimensional over k, V is a simple D-bimodule where k acts centrally and $\mathrm{dim_D(V)} = 1$.