1. Cohen–Macaulay modules over Gorenstein surface singularites

A *surface singularity* A means here the completion of the local ring of a singular point of an algebraic surface over an algebraically closed field. We denote by $\pi : X \to S = \operatorname{Spec} A$ the *minimal resolution* of this singularity and by E the *exceptional curve* on X, i.e the preimage of the unique closed point of S. Let $\mathrm{CM}(A)$ be the category of (maximal) Cohen–Macaulay A-modules. We suppose that A is *normal* (i.e. an integrally closed domain) and *Gorenstein*, i.e. A is Ext-injective in the category $\mathrm{CM}(A)$. It is known, due to Herzog, Artin–Verdier, Esnault, Auslander, that A is *Cohen–Macaulay finite*, i.e. has only finitely many non-isomorphic indecomposable Cohen–Macaulay modules, if and only if it is a *rational double point* or, the same, the singularity of *Dynkin type A-D-E* in the sense of Arnold. Kahn, Greuel and the author have proved that if A is a *simple elliptic* or a *cusp* singularity, it is *Cohen–Macaulay tame*, i.e. non-isomorphic indecomposable Cohen–Macaulay modules of any prescribed rank form at most 1-parameter families. We prove that in all other cases A is *Cohen–Macaulay wild*. As corollaries, we get analogous results for *normal* $\mathbb{Q}$-*Gorenstein* surface singularities, i.e. such that the canonical divisor K_A is of finite period in the group of divisor classes, as well as for *isolated hypersurface singularities* (of any dimension).