1. Cohen–Macaulay modules over Gorenstein surface singularités

A surface singularity $A$ means here the completion of the local ring of a singular point of an algebraic surface over an algebraically closed field. We denote by $\pi : X \to S = \text{Spec } A$ the minimal resolution of this singularity and by $E$ the exceptional curve on $X$, i.e. the preimage of the unique closed point of $S$. Let CM($A$) be the category of (maximal) Cohen–Macaulay $A$-modules. We suppose that $A$ is normal (i.e. an integrally closed domain) and Gorenstein, i.e. $A$ is Ext-injective in the category CM($A$). It is known, due to Herzog, Artin–Verdier, Esnault, Auslander, that $A$ is Cohen–Macaulay finite, i.e. has only finitely many non-isomorphic indecomposable Cohen–Macaulay modules, if and only if it is a rational double point or, the same, the singularity of Dynkin type $A$-$D$-$E$ in the sense of Arnold. Kahn, Greuel and the author have proved that if $A$ is a simple elliptic or a cusp singularity, it is Cohen–Macaulay tame, i.e. non-isomorphic indecomposable Cohen–Macaulay modules of any prescribed rank form at most 1-parameter families. We prove that in all other cases $A$ is Cohen–Macaulay wild. As corollaries, we get analogous results for normal $\mathbb{Q}$-Gorenstein surface singularities, i.e. such that the canonical divisor $K_A$ is of finite period in the group of divisor classes, as well as for isolated hypersurface singularities (of any dimension).