COMMENTS ON

# AN ALTERNATIVE APPROACH TO TILTING THEORY

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(1) **Motivation, page 5**

In module theory, for a module ${}_RP$ and $E = \mathrm{End}(P)$, the functors

$$P \otimes_E \mathrm{Hom}(P,-) : {}_R\mathbb{M} \to {}_R\mathbb{M} \text{ and } \mathrm{Hom}(P, P \otimes_E -) : {}_E\mathbb{M} \to {}_E\mathbb{M}$$

are permanently used but ignoring that these endofunctors have an interesting structure as comonad and monad, respectively. We want to find out what category theory can contribute to tilting theory, for example.

(2) **Adjoint functors, monads, comonads, page 6 - 11**

This is an overview of the elementary categorical notions needed. The focus on the subcategory $\mathsf{Reg}(S,\varepsilon)$ seems to be new.

(3) **Idempotent comonads, page 12 - 16**

These comonads play a central role here, they lead to a chain of comonad morphisms (going back to Fakir) and a possible definition of "comonadic length" for objects.

(4) **Adjunctions, page 16 - 23**

(i) First the (new) subcategories are reconsidered.

(ii) The comparison functor is defined; it is fully faithful if and only if $\mathbb{B} = \mathsf{Reg}(FG,\varepsilon)$; for the existence of a left adjoint, cocompleteness conditions on $\mathbb{B}$ are required.

(iii) The properties of the new adjunction $F_{[1]} \dashv G_{[1]}$ depend on the (exactness) properties of $G_{[1]}$.

(iv) $F_{[1]}G_{[1]}$ is the coequalising functor of $FG$ - this can be connected with the monadic decomposition of the functor $G$

(5) **Relative homological aspects, page 24 - 32**

For an adjunction $F \dashv G$, the class $FG(\mathbb{B}) := \{FG(B)\,|B \in \mathbb{B}\}$ as the class of projectives in the category $\mathbb{B}$. This is a *precovering* class and in case $\mathbb{B}$ is preadditive with kernels, it is *covering* if and only if $FG$ is idempotent. The resulting relative projective resolutions are represented by the *standard resolution* for the comonad $(FG, F\eta G, \varepsilon)$.

(6) $\mathbb{B}(P,P)$**-modules versus** $\mathbb{B}(B,-)$**-modules, page 33 - 35**

We consider abelian categories $\mathbb{B}$ and $P \in \mathbb{B}$ and a monad morphism $\varphi : \mathbb{B}(P,P)- \to \mathbb{B}(B,-) =: T$ with a functor $\mathsf{Ab}^{\varphi} : \mathsf{Ab}_T \to {}_{\mathbb{B}(P,P)}\mathsf{Ab}$. Notice that the lower part of the right diagram on page 34 need not be commutative (unless $\varphi$ is an isomorphism).