Teaching Statement

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To me, mathematics on the most basic level is about thinking and rational reasoning. As a teacher my primary goal is always for my students to think first, instead of looking for some "magical formula" for the solution. The skill of thinking needs to be complemented by the ability of explaining both verbally and in writing. I believe that by mastering these, students at any level will gain something from the class, regardless of their future career.

Using mathematical language and understanding its formalism is necessary for communicating our ideas, but I feel that for most people it is not the right way to actually think about mathematics. Everyone should develop their own intuition about mathematical concepts, and it is the teacher's job to help this by emphasizing intuition first, and then showing how the formalism reflects the picture we have in our mind.

Small groups

The most rewarding experience in mathematics is the moment of discovery. I think this is so fundamental, that anything that you figure out yourself will be absorbed much better than if it is only explained to you. Hence it is very important to allow time for the students to make these discoveries. The best setting for this is a small class where students work on problems in the classroom.

I always try to build problem sets with exercises ranging from routine to very challenging so that everybody has some success but nobody gets bored. The small number of participants allows me to survey the progress of each student as they work on the problems. If someone gets stuck, I am ready to help, but never by showing the next step, rather by asking questions to redirect their thinking.

I am also a big proponent of the "experimental method". Cut a sheet along \( n \) straight lines. At most how many pieces of paper will you end up with? I have found that getting started is often the hardest part of solving such an abstract problem. It is very helpful to experiment with concrete instances of the abstract setting: draw zero, one, two, three, four lines, and count the number of pieces; find the pattern between the answers for the concrete cases, and come up with a conjecture on how to attack the original problem. In our example the number of pieces would be 1,2,4,7,11. Once you realize that the difference grows by one each time, the problem suddenly becomes much more tractable.

The discussion of problems is a great opportunity to let students practice their explaining skills. But having to talk at the blackboard can be uncomfortable for the student or boring for the class. I have found a solution that works very well in my experience. I have the students explain their solutions from their seats while I try to take notes on the board of what they are saying, just as if they were teaching me. By taking on the role of someone who has no a priori knowledge of the problem, I am able to solicit very clear explanations from the students.

I have been able to refine and experiment with these approaches throughout my years at college and in grad school. Already in my sophomore year, I was invited to be a section leader for the Discrete Mathematics course, an assignment I kept until the end of my phd studies. Later I led sections for courses in various levels of Calculus and Topology too. The way courses are run at our university, sections are a vital part of the curriculum. In a section, a group of around 10-15 students meet weekly to deepen their understanding of the course material by solving related problems. As a section leader I was responsible for assembling (and often inventing) the exercises, supervising students while they work on the problems in class, and finally discussing the possible solutions. Thus I have been able to master the most important skills for problem based teaching in small groups.
Lectures

I started teaching in a lecture based format when I was appointed to teach a full course at the Budapest Semesters in Mathematics (BSM)\footnote{http://www.budapestsemesters.com/}, one of the most prestigious study-abroad programs for American and Canadian undergraduate students. BSM hires its teachers on a semester-to-semester basis, striving to provide its students with the best possible learning experience. I taught Combinatorics for two semesters and Topology for three semesters. Here I had full responsibility for the courses, from writing syllabi (along given guidelines but with much flexibility) to grading homework, exams, and assigning final grades. I have also had the honor of teaching a graduate course at Harvard in 2011.

Though problem solving sessions are the ideal format for discovery based learning, in my opinion lectures should also be geared towards this goal as much as possible. I think the best way to achieve this is to encourage an interactive environment during lectures. I always allow students to interrupt by asking questions, but more importantly I frequently stop in the middle of an argument and let the class think about how to go on. This often leads to open discussion where many students share their ideas until the joint effort succeeds. Doing this regularly gives me constant feedback on how the class is digesting the material, so I can easily adjust the speed or even go back and repeat things if necessary.

I always try to keep my lectures from becoming too dry. Formulas are a concise way of writing definitions and proofs, but they rarely convey the real meaning behind the concepts. My goal in lectures is to encourage the students to think in pictures and ideas first. Even as simple tricks as drawing a circle when talking about a set or representing a sequence with dots instead of \( x_n \) work surprisingly well. When teaching new material, I find it very effective to first explain things in a highly intuitive way even if this initially takes away from the precision. Once the students understand the idea, it is much easier for them to translate it into something formal and precise, to remember it, and later to apply it.

Building my own courses

So far, I have had the opportunity to design and teach two courses from scratch. I taught "Amenability" in Budapest and "Graph Limits" at Harvard.

I believe that a well-designed course should be useful and rewarding: it should expand the students’ knowledge by adding important new techniques and ideas, but there has to be sufficient motivation for working through the technical details. This could either be the proof of some famous theorem in an advanced course, or interesting real-life applications in an introductory course.

In 2009 I was invited by the Eotvos Collegium, a prestigious honor society of the Eotvos University, to teach a course in their mathematics lab. Given complete freedom, the biggest challenge was finding a suitable topic. It had to be intriguing and at the same time accessible to math majors with limited background. I chose amenable groups to be in the focus. Amenability is a central concept in contemporary mathematics that will be useful for any aspiring mathematician. The Banach-Tarski paradox and related results served the purpose of keeping the students interested. To me, this is a prime example of how the interplay of seemingly unrelated fields can lead to surprising results, and as such shows how organic and beautiful math is.

I used the same principles when planning the graduate-level research topics course I taught at Harvard. I was asked to teach graph limits, a very recent field of research with lot of technical notions and diverse results. I managed to keep the course coherent and useful by choosing a technique (ultra products) as a governing theme. This technique originated in logic and model theory, but has since found applications in almost every field of mathematics, so students definitely gained something lasting by taking the course. Finally, to motivate students to work through all the technical details, I was able to gear the whole course towards the proof of the famous theorem of Szemerédi on arithmetic progressions, for which he recently received the Abel prize.

Vision

As a child I participated in the math camps run by Lajos Pósa\footnote{http://en.wikipedia.org/wiki/Lajos_Posa_(mathematician)}, probably the most famous Hungarian mathematics educator. Starting from my junior year in high school, I had the honor of being his assistant in these camps for several years. His passion, and his philosophy of learning through discovery had a lasting impact on how I approach teaching. My mission is to spread his legacy and raise mathematically open minded students.