QUANTIZED QUIVER VARIETIES

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Nakajima quiver varieties are symplectic algebraic varieties that are moduli spaces for certain quiver representations. They were introduced by Nakajima in order to provide geometric realizations of representations of Kac-Moody Lie algebras and related objects. Examples include cotangent bundles to partial type A flag varieties, minimal resolutions of Kleinian singularities, their Hilbert schemes of points, moduli spaces of torsion free sheaves. One nice feature of the Nakajima quiver varieties is that they are symplectic resolutions of singularities, an especially nice class of crepant resolutions popular in Algebraic Geometry.

One thing that a representation theorist can do with a symplectic variety is to quantize it getting an associative algebra (or a sheaf of algebras). In the case of symplectic resolutions, following Bezrukavnikov and Okounkov, one expects tight connections between the representation theory of quantizations and the geometry of resolutions. I am going to focus on the quantizations of the quiver varieties. The resulting algebras include several algebras studied previously such as universal enveloping algebras and W-algebras of type A, most interesting symplectic reflection algebras of Etingof and Ginzburg, as well as interesting new algebras.

In my lectures, I am going to give an introduction to the geometry of quiver varieties and the representation theory of their quantizations emphasizing connections between the two subjects. In particular, I will explain a solution for a basic problem regarding the representation theory of the quantizations: to compute the number of finite dimensional irreducible representations. This is based on a partly joint work with Bezrukavnikov.

Preliminary plan.

1) *Nakajima quiver varieties.* Introduction to Geometric Invariant theory, moment maps and Hamiltonian reductions. Quivers and their representations. Definition, examples and basic properties of Nakajima quiver varieties.

2) *Quantizations.* Quantizations of algebras. Quantum Hamiltonian reductions. Examples. Quantizations of non-affine varieties.

3) *Modules over quantizations.* Definition and basic properties. Localization theorems. Kac-Moody algebras and Nakajima's geometric construction of their representations. The main result of the number of finite dimensional irreducible representations of quantized quiver varieties.

4) *Lower bound.* Categorical $\mathfrak{sl}_2$-actions. Basics on D-modules. Webster's categorification of Nakajima's construction. Isomorphisms between quiver varieties and their quantizations. The sketch of a proof of the lower bound.

5) *Upper bound.* Wall-crossing functors. Harish-Chandra bimodules. Simple wall-crossing functors and a reduction to rank 1. Wall-crossing functors in rank 1 cases.