RCA, PROBLEM SET 6

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0.1. Let $c = \frac{a}{d}$, where $d > 1, a > 0, \mathrm{GCD}(a, d) = 1$. Show that $\mathrm{Ind}_{S_{md}}^{S_d^m} L_c((d))^{\boxtimes m}$ decomposes as $\bigoplus_{\tau \vdash m} L_c(d\tau)^{\oplus \dim \tau}$, where by $\dim \tau$ we mean the dimension of the corresponding representation.

0.2. Let $\mathfrak{g}$ be a Lie algebra and $\mathfrak{g}_1, \mathfrak{g}_2$ be subalgebras of $\mathfrak{g}$ such that $\mathfrak{g} = \mathfrak{g}_1 + \mathfrak{g}_2$. Show that, for a $\mathfrak{g}_1$-module, there is a natural isomorphism

$$\mathrm{Ind}_{\mathfrak{g}}^{\mathfrak{g}_1}(V)/\mathfrak{g}_2 \,\mathrm{Ind}_{\mathfrak{g}}^{\mathfrak{g}_1}(V) \cong V/(\mathfrak{g}_1 \cap \mathfrak{g}_2)V.$$

0.3. Show that $\langle M_0, M_\infty | \{0, \infty\} \rangle \cong \mathrm{Hom}_{\hat{\mathfrak{g}}}(M_0, \mathbb{D}M_1)^*$.