RCA, PROBLEM SET 5

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0.1. Let $\mathcal{C}, \mathcal{C}', \mathcal{D}, \mathcal{D}'$ be abelian categories equivalent to categories of modules over finite dimensional associative algebras over a base field $\mathbb{F}$. Let $\varphi_{\mathcal{C}} : \mathcal{C} \to \mathcal{C}', \varphi_{\mathcal{D}} : \mathcal{D} \to \mathcal{D}'$ be exact functors and let $\pi : \mathcal{C} \to \mathcal{D}, \pi' : \mathcal{C}' \to \mathcal{D}'$ be quotient functors. Suppose that

- $\pi' \circ \varphi_{\mathcal{C}} \cong \varphi_{\mathcal{D}} \circ \pi$,
- π, π' are fully faithful on the projective objects,
- $\varphi_{\mathcal{C}}, \varphi_{\mathcal{D}}$ map the projective objects to the projective objects.

Show that $\operatorname{End}(\varphi_{\mathcal{C}}) = \operatorname{End}(\varphi_{\mathcal{D}})$. Moreover, check that φ_C is uniquely recovered from the remaining three functors.

0.2. Let $W'' \subset W' \subset W$ be parabolic subgroups. Then ${}^{\mathcal{O}}\operatorname{Res}_W^{W''} \cong {}^{\mathcal{O}}\operatorname{Res}_{W'}^{W''} \circ {}^{\mathcal{O}}\operatorname{Res}_W^{W'}$.

0.3. For $c = \frac{a}{d}$ with $a > 0$ and $\operatorname{GCD}(a, d) = 1$ prove the following identity in $K_0(\mathcal{O}_c(kd))$:

$$\sum_{i=0}^{k-1} (-1)^i [L_c((k-i)d, d^i)] = \sum_{j=0}^{dk-1} (-1)^j [\Delta_c(dk - j, 1^j)]$$