0.1. Check that the formulas
\[ x \mapsto x, w \mapsto w, y \mapsto D_y := \partial_y + \sum_{s \in S} \frac{c(s)(\alpha_s, y)}{\alpha_s} (s - 1), \quad x \in h^*, w \in W, y \in h, \]
define an algebra homomorphism \( H_c(W, h) \to D(h^{reg})#W \) (e.g., following the sketch given in the lecture).

0.2. Check that the Euler element \( h = \sum_{i=1}^n x_i y_i - \sum_{s \in S} c(s)s \) satisfies
\[ [h, x] = x, [h, w] = 0, [h, y] = -y. \]

0.3. Let \( C \) be a highest weight category. Show that the standard object \( \Delta_L \) is the projective cover of \( L \) in the Serre subcategory of \( C \) spanned by \( L' \leq L \).

0.4. Show that in \( O_c \) we have the following (we don’t know at this point that \( O_c \) is a highest weight category):
- \( \text{Hom}(\Delta_c(\tau), \Delta_c(\tau')) \neq 0 \Rightarrow \tau \leq_c \tau' \),
- \( \text{End}(\Delta_c(\tau)) = \mathbb{C} \),
- \( \text{Ext}^1(\Delta_c(\tau), \Delta_c(\tau')) \neq 0 \Rightarrow \tau <_c \tau' \).

0.5. Prove that the map \([\tau] \mapsto [\Delta_c(\tau)]\) is an isomorphism \( K_0(W\text{-rep}) \xrightarrow{\sim} K_0(O_c) \). Furthermore, prove that two objects in \( O_c \) with the same character have the same classes in \( K_0(O_c) \).

0.6. Let \( M_1, M_2 \in O_c \) be such that \( M_1 \oplus M_2 \) has a Verma filtration. Show that both \( M_1, M_2 \) have Verma filtrations (again, at this point we still don’t know that \( O_c \) is highest weight).

0.7. Let a parameter \( c \) be generic in the sense that \( \tau \leq_c \tau' \) implies \( \tau = \tau' \). Show that the category \( O_c \) is semisimple and that \( P_c(\tau) = \Delta_c(\tau) = L_c(\tau) \).