On Bubbles of Nothing and Merging the Swampland Conjectures

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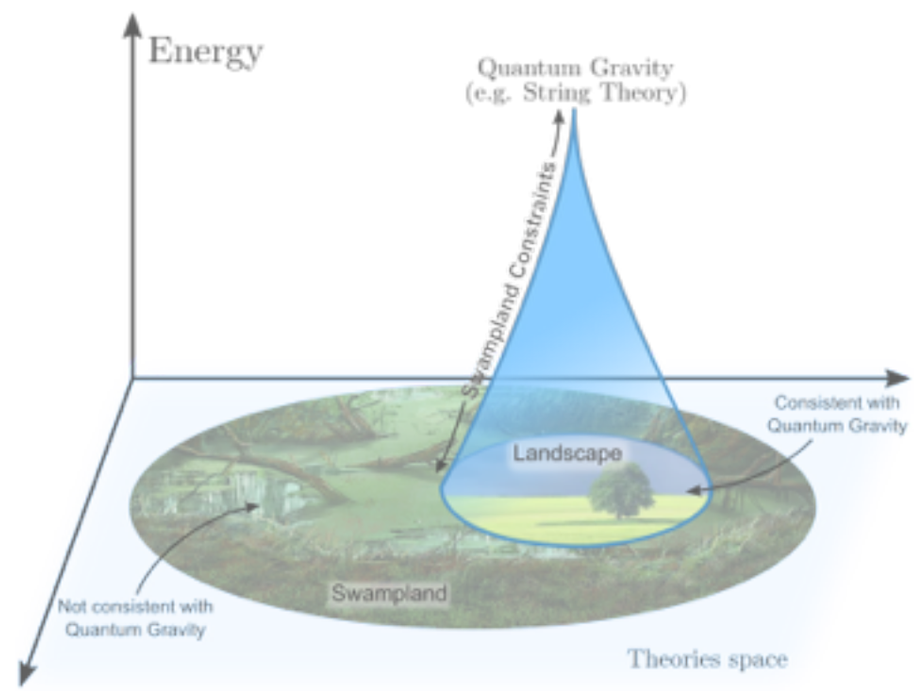


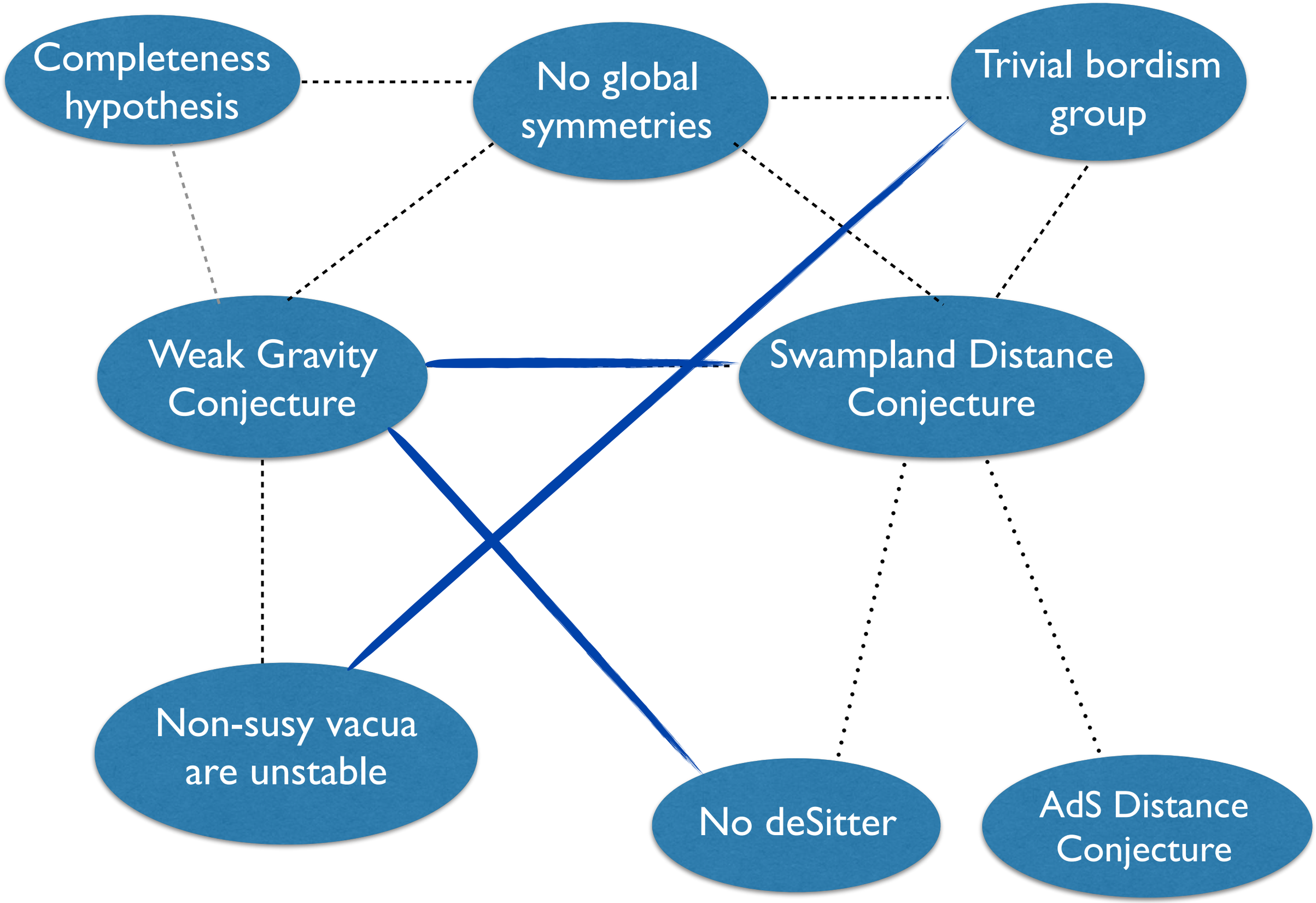
2004.10768 with Gendler

2005.06494 with Garcia-Etxebarria,Montero,Sousa

2006.xxxxx with Lanza,Marchesano,Martucci

StringPheno, June 2020

Swampland Conjectures



Outline:

(1) Nothing is certain in string compactifications

2005.06494 with I.Garcia-Etxebarria, M.Montero, K.Sousa

(2) Merging WGC and SDC using BPS states

2004.10768 with N.Gendler

(3) Swampland conjectures for strings and membranes

2006.xxxxx with S.Lanza, F.Marchesano, L.Martucci

(1) Nothing is certain

Are non-susy vacua always unstable?

- Swampland conjecture [Ooguri-Vafa'16]
  [Freivogel,Kleban'16]

- Quest for universal instability: bubbles of nothing

Bubbles of nothing

Non-perturbative instability from the vacuum to nothing!

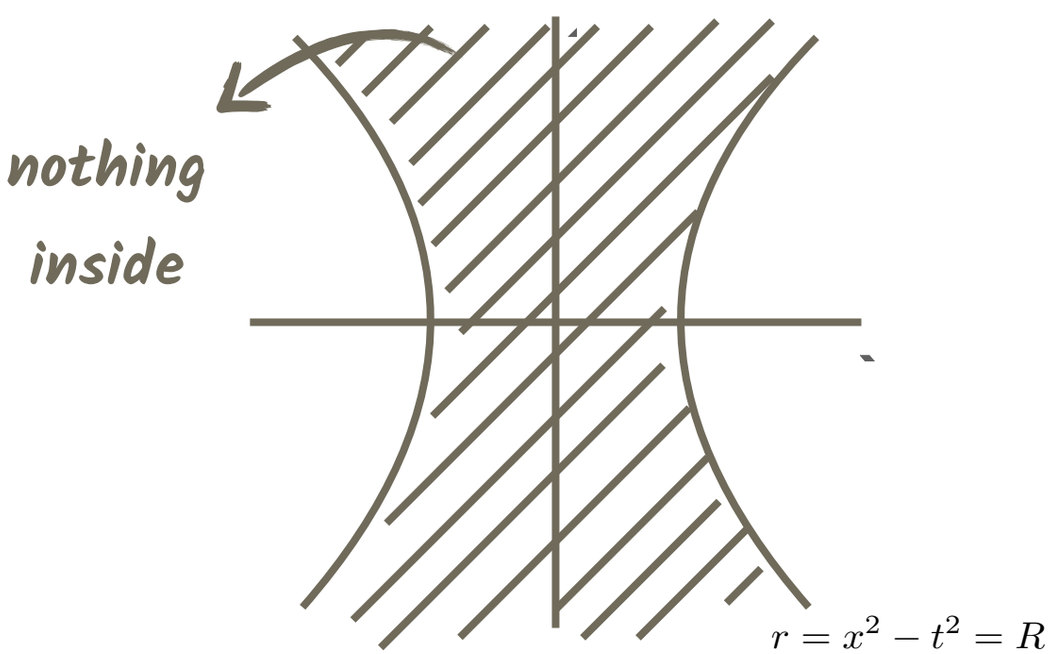
Consider $M_4 \times S^1$ [Witten'81]

Instanton solution: $ds_5^2 = r^2 d\Omega_3^2 + \dfrac{dr^2}{1 - R^2/r^2} + R^2 \left(1 - \dfrac{R^2}{r^2}\right) d\theta^2 \qquad (r > R)$

Endpoint of decay: $ds_5^2 \underset{r\to\infty}{\approx} -dt^2 + dx^2 + x^2 d\Omega_2^2 + R^2 d\theta^2.$

$x = r\cosh\psi, t = r\sinh\psi$

Minkowski with a hole!



- Spacetime ends at $r = R$ (bubble wall)
- Size of circle: $R\sqrt{1 - R^2/r^2}$

  (shrinks to zero size at bubble wall)

→ non-singular

Bubbles of nothing

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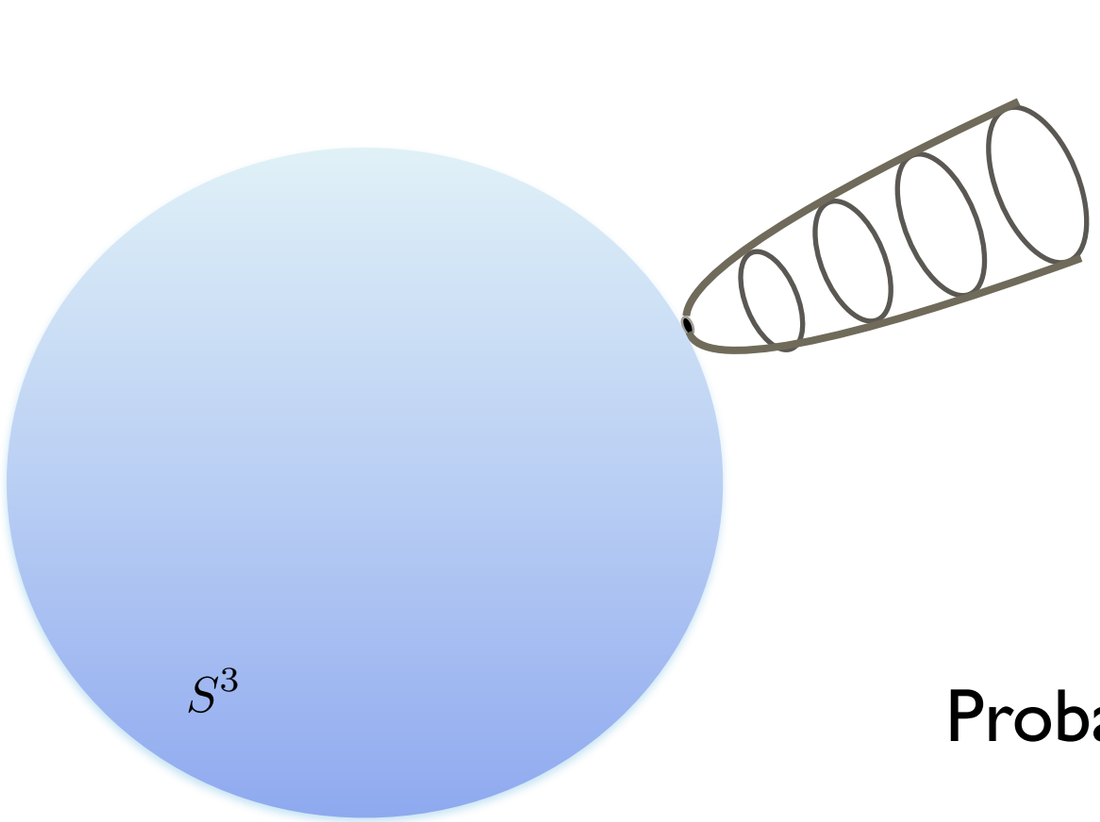


Topology:

$$S^3 \times Disk$$

Probability of decay:

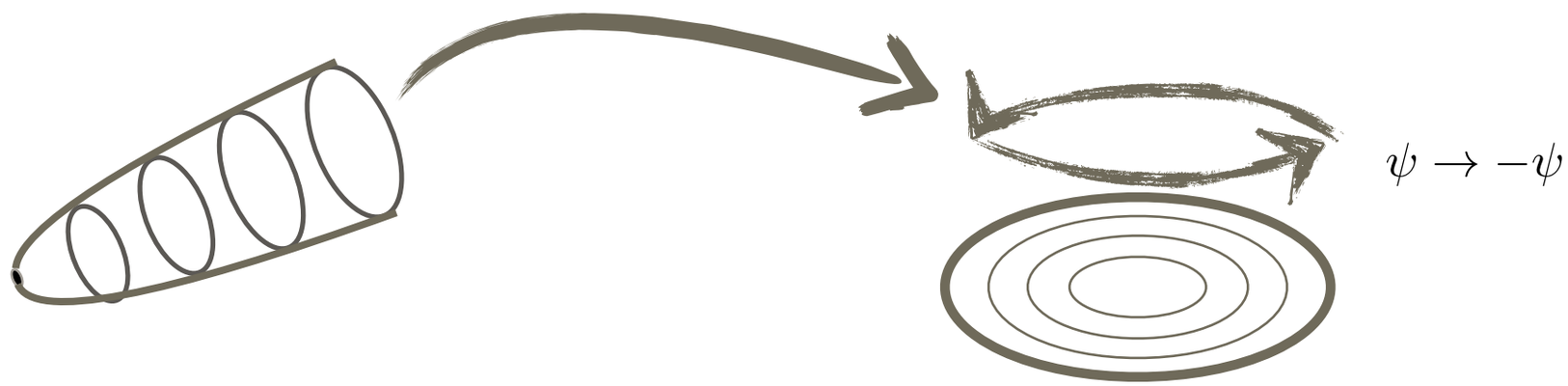
$$\Gamma = e^{-S} \quad ; \quad S = \frac{\pi R^2}{8G_4}$$

Bubbles of nothing

Shall we worry?

Topological obstruction: [Witten'81]

Fermions must be well defined in the disk: antiperiodic b.c.



Bubble does not exist if fermions have periodic (SUSY) boundary conditions

Lesson: **WRONG!** (see also [Blanco Pillado et al'16])

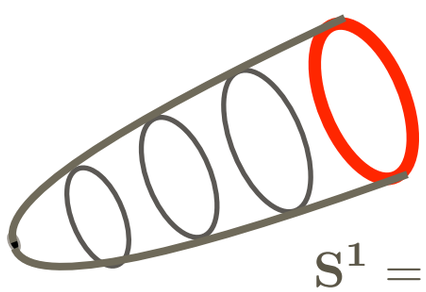
Bubbles will not arise in SUSY theories (even if vacuum is non-SUSY)

Topological obstruction

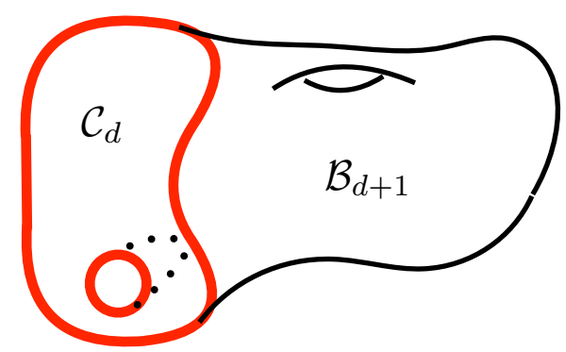
Consider $M_{D-d} \times C_d$ compact manifold

BON exists if C_d can shrink to a point $\Longleftrightarrow$ $C_d = \partial B_{d+1}$

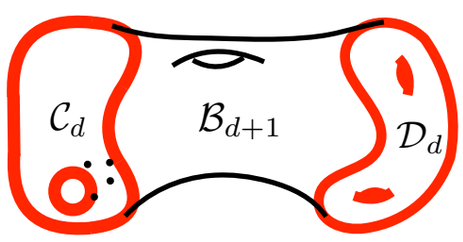
and the spin structure of C_d extends to B_{d+1} (must be a boundary)



$\mathbf{S}^1 = \partial \mathbf{D}$



When $C_d = \partial B_{d+1}$? If it belongs to trivial class on Ω_d^{spin} bordism group



$\mathcal{C}_d \sim \mathcal{D}_d$

Topological obstruction

$d = 1$ → $\Omega_1^{\text{spin}} = \mathbb{Z}_2$

- Non-trivial class: periodic bc → No BON
- Trivial class: antiperiodic bc → BON

$d = 2$ → $\Omega_2^{\text{spin}} = \mathbb{Z}_2$ Same as above

$d = 3$ → $\Omega_3^{\text{spin}} = 0$

Any three dimensional manifold admits a BON!

(even with SUSY preserving b.c.)

d	0	1	2	3	4	5	6	7	8	9
Ω_d^{spin}	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	$2\mathbb{Z}_2$

- M-theory on G2
- 10D String theory to 4D (any CY!)
- AdS compactifications on 5-manifolds

Topological obstruction

In general $\Omega_d^{\#}$ → *depends on charged fermions, fluxes...*

[McNamara, Vafa'19]

Swampland conjecture: $\Omega_d^{\mathrm{QG}} = 0$ (to avoid global symmetries)

Consequence: No topological obstruction to construct bubbles of nothing in Quantum Gravity

We will focus on $\Omega_3^{\mathrm{spin}} = 0$

Dynamical obstruction

Any three dimensional spin manifold admits a bubble
(including T^3 with SUSY-preserving boundary conditions)

What prevents SUSY vacua from decaying?

Only energetically favourable if there is an asymptotically flat solution with $M_{\rm ADM} = 0$ different from the vacuum.

Otherwise, CdL suppression: $R \to \infty \ , \ \Gamma \sim e^{-S} \to 0$

Positive Energy Theorem: $M_{\rm ADM} > 0$ if: [Witten'82]

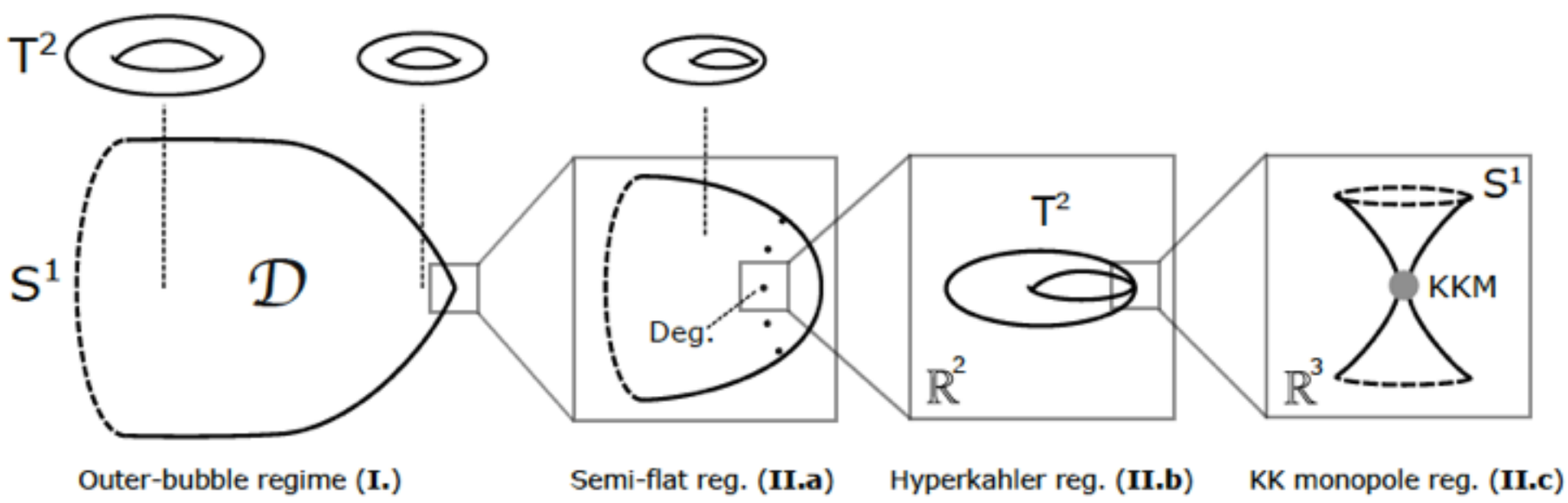
- Asymptotically covariantly constant spinors
- Satisfy Dominant Energy Condition (DEC): $T_{00} > 0 \ ; \ T_{00}^2 > \sum_k T_{0k} T^{0K}$

New BON

We construct an explicit bubble solution for T^3 with SUSY b.c.

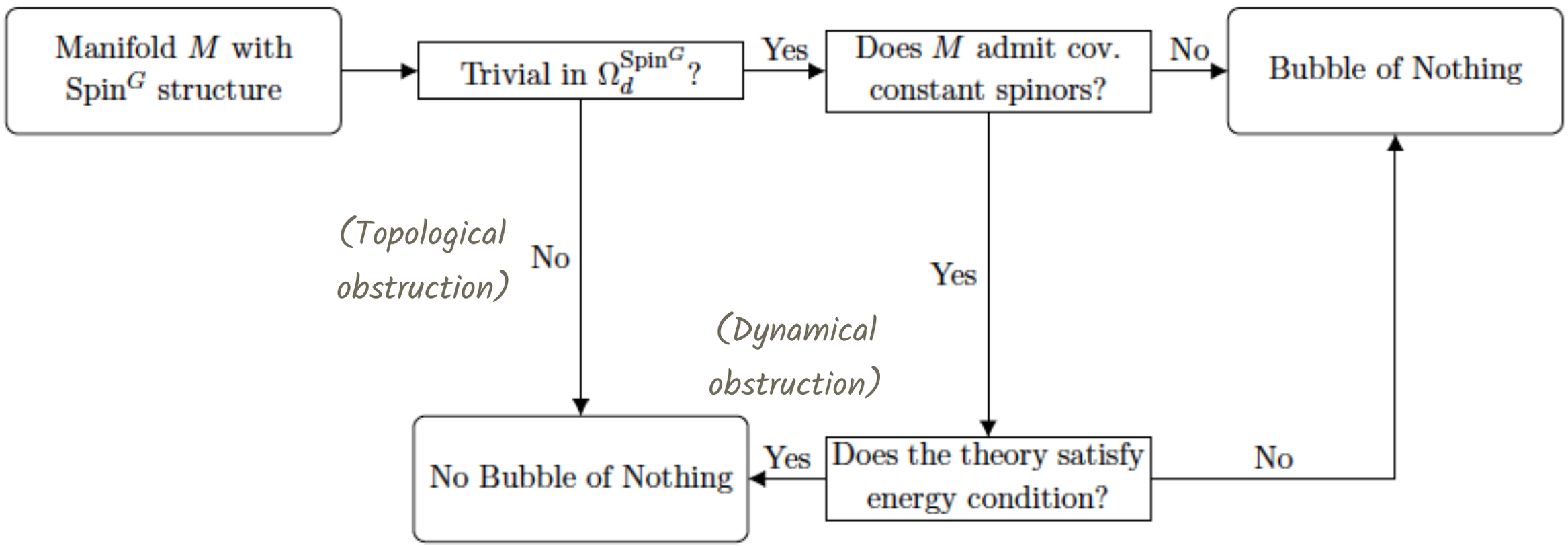
We violate DEC (and break SUSY) by a Gauss-Bonnet term:

$$\mathcal{L} = e^{-2\phi}(R + 4(\Delta\phi)^2 - \frac{1}{12}H^2 - \frac{\alpha'^2}{8}R_{GB}^2)$$



$$R_{BON} \sim \left(\frac{24\pi^2}{\mathcal{V}_{T^3}}\alpha'\right)^{-1} \qquad S_{BON} \sim \left(\frac{24\pi^2}{\mathcal{V}_{T^3}}\alpha'\right)^{-(D-5)}$$

Recipe



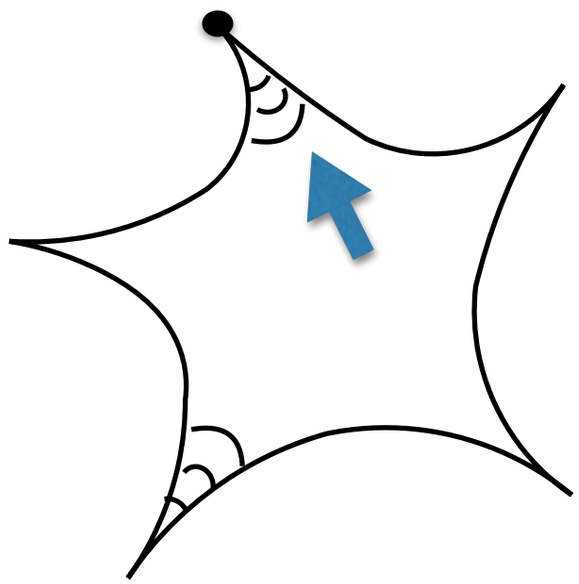
- DEC generically violated when SUSY is broken by quantum effects/Casimir energies, higher derivative terms…

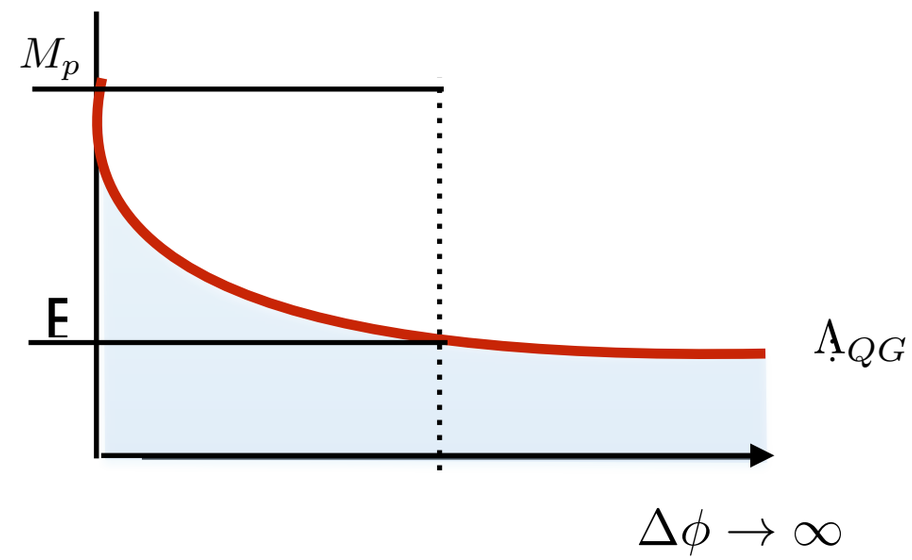
- Energy condition modified with fluxes: $T_{00}^2 > \sum_k T_{0k} T^{0K} + \sum_{ij} \frac{q^2}{(8\pi G)^2} F_{ij} F^{ij}$

WGC?

(2) Merging WGC and SDC in N=2

Asymptotic limits in moduli spaces





These limits seem under control from the point of view of QFT

but still, the EFT must break down when approaching the boundary
by quantum gravity effects

Approximate global symmetries,
Weakly coupled gauge theories,
Large field ranges…

…come at a price.

Swampland conjectures:

- Infinite towers of massless states WGC, SDC
- Runaway potentials dSC

Asymptotic limits in susy moduli spaces

Use asymptotic Hodge theory to study the physics at the asymptotic limits in a model-independent way

[Grimm, Palti, IV'18] [Grimm,Palti,Li'18] [Corvilain, Grimm, IV'18] [Grimm,van de Heisteeg'19] [Grimm,Li, IV'19]

see also [Cecotti'20]

What does asymptotic mean? Some scalar s takes large values

Asymptotic limit: regime at which perturbative expansion in parameter $1/s$ makes sense

e.g large volume, weak coupling...

Swampland Distance Conjecture: $m \sim m_0 e^{-\lambda \Delta\phi}$ when $\Delta\phi \to \infty$

Important question: What is λ ?

Bounds on λ

(Calabi-Yau N=2 moduli spaces)

We find: λ is related to the properties of a discrete infinite symmetry generating the tower of BPS states (wrapping branes)

$$\lambda = \left| \frac{\nabla_i M}{M} u_i \right| \geq \frac{1}{\sqrt{2d}}$$

- One field [Grimm, Palti, IV'18]
- Multi-field [Gendler, IV' 20]

unit vector along geodesic

$d \equiv$ ***integer associated to discrete symmetry***

(it characterises the type of asymptotic limit)

For Calabi-Yau threefold compactifications: $d = 1, 2, 3$

$$\lambda \geq \frac{1}{\sqrt{6}}$$

Any meaning?

Relation to WGC

If there is a gauge coupling vanishing at the infinite field distance limit:

Same tower satisfies both WGC and SDC and…

… the exponential rate λ is fixed by extremality bound of black holes!

[Grimm, Palti, IV'18] [Lee,Lerche,Weigand'18-20] [Gendler, IV' 20]

Relation to TCC: [Andriot et al'20]

D=4

$$\lambda = \frac{1}{\sqrt{(D-1)(D-2)}} = \frac{1}{\sqrt{6}}$$

But the coincidence is lost in higher dimensions…

WGC with scalar fields

- Extremality bound:

If gauge kinetic function $\mathcal{F} \sim e^{-\alpha_i \phi^i}$

$$\frac{\mathcal{Q}}{M} \geq \left(\frac{\mathcal{Q}}{M}\right)\Bigg|_{\text{extremal}} = \frac{D-3}{D-2} + \frac{1}{4}|\vec{\alpha}|^2$$

- Repulsive force condition: [Palti'16], [Heidenreich,Reece,Rudelius'19]

$$\left(\frac{\mathcal{Q}}{M}\right)^2 \geq \frac{D-3}{D-2} + \frac{g^{ij}\partial_i M \partial_j M}{M^2}$$

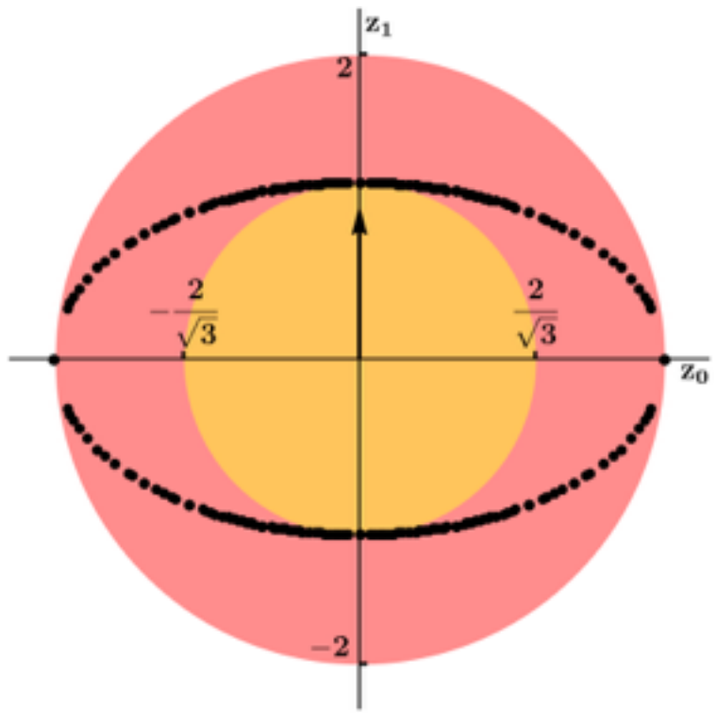
They seem to coincide at the weak coupling limits, implying

SDC factor: $\lambda = \frac{\nabla M}{M} = \frac{\alpha}{2}$ [Lee,Lerche,Weigand'18]

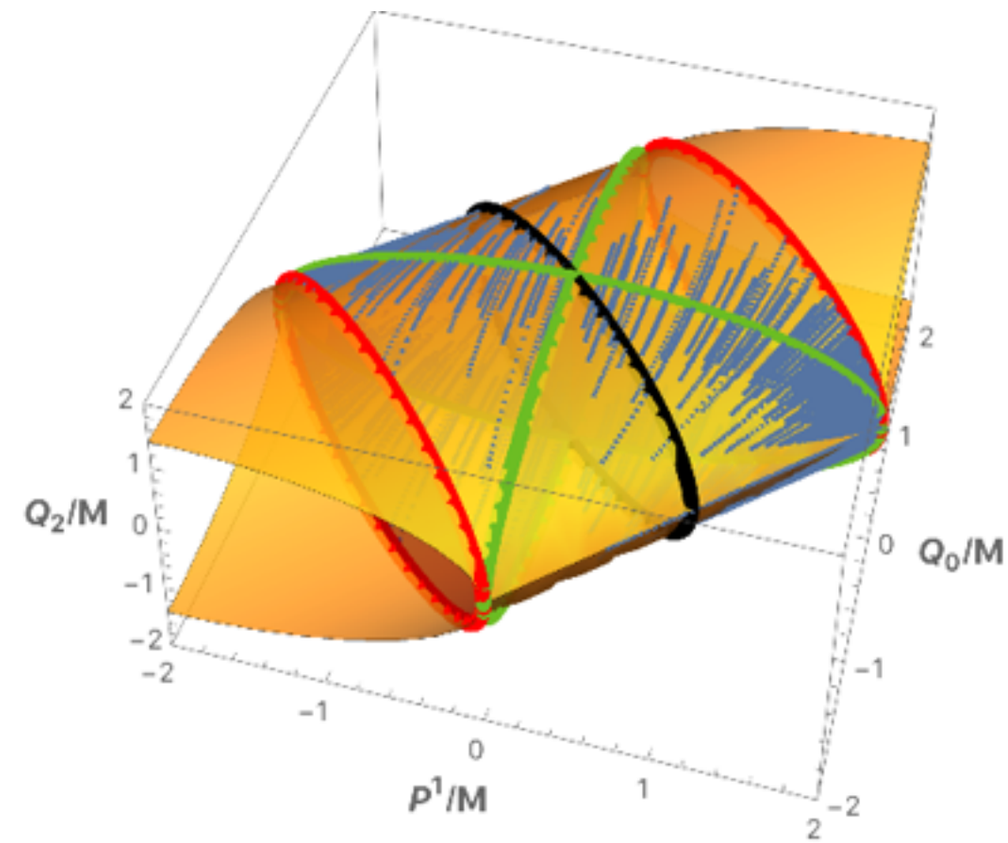
[Gendler, IV' 20]

WGC and BPS states in N=2

Charge to mass ratio of BPS states:



1 modulus



2 moduli

Principal radii of the ellipse:

$$\gamma_1^{-2} = \sum_{i|\mathrm{Im}(\Pi_0^{E,I})=0} \left(\frac{Q}{M}\right)\Big|_{q_E^I}^{-2} \; ; \; \gamma_2^{-2} = \sum_{i|\mathrm{Im}(\Pi_0^{E,I})\neq 0} \left(\frac{Q}{M}\right)\Big|_{q_E^I}^{-2}$$

WGC and BPS states in N=2

We can provide principle radii at any asymptotic limit of field space:

$$\left(\frac{Q}{M}\right)\Big|_{q^I}^2 = 1 + \underbrace{\sum_{i=1}^{n} \frac{(\ell_i^I - \ell_{i-1}^I)^2}{d_i - d_{i-1}}}_{\frac{\nabla_i M}{M} = \frac{\alpha_i}{2}}$$

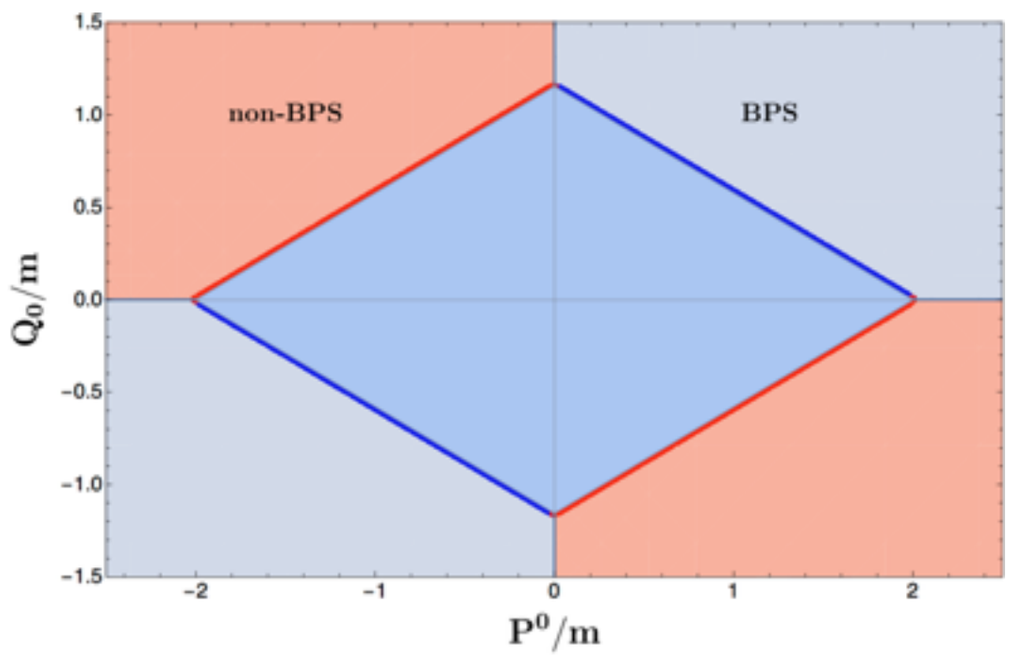
discrete data associated to the singular limit

BPS states are extremal at the asymptotic regimes ("WGC=RFC")

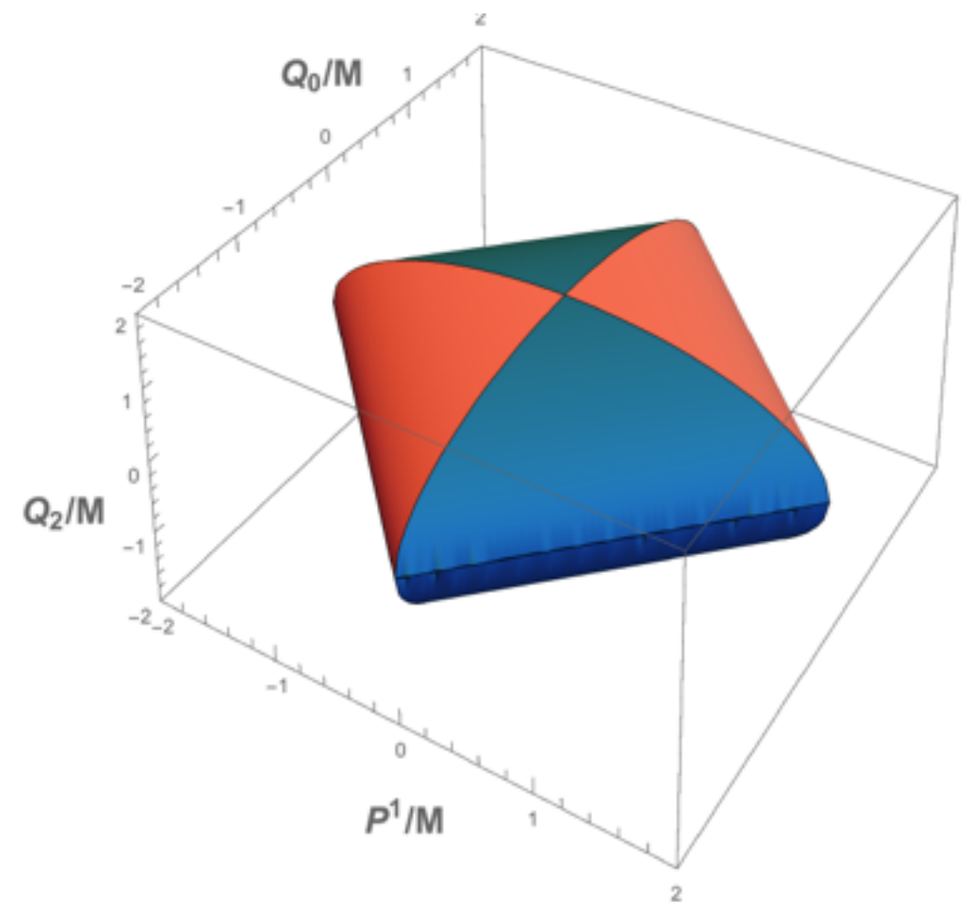
SDC = Tower WGC → $\lambda = \left|\frac{\nabla_i M}{M} u_i\right| \geq \frac{1}{\sqrt{2d}}$

WGC and BPS states in N=2

Extremalility bound along non-BPS directions:



1 modulus



2 moduli

In these examples, BPS states are enough to satisfy WGC convex hull condition

Merging WGC and SDC

At every infinite field distance limit with a gauge coupling going to zero, there is an infinite tower of states with

$$\frac{Q}{M}^2 \geq \frac{Q}{M}^2\Big|_{extremal} = \frac{D-3}{D-2} + \frac{g^{ij}\partial_i M \partial_j M}{M^2}$$

and the gauge coupling decreases exponentially with the geodesic field distance.

SDC= Tower WGC → λ fixed by extremality bound

No more O(1) unknown factors

Merging WGC and SDC

Is there always a vanishing gauge coupling at infinite distance?

(also works for a KK tower)

➡ It seems so if one allows for a p-form gauge coupling

Similar to string emergence conjecture [Lee,Lerche,Weigand'19]

- Decompactification
- String pert. limit

(3) Swampland for strings and membranes

Swampland conjectures for strings and membranes

WGC in 4D:

p	Gauge field	State
0	Axion ϕ	Instanton
1	Gauge field A_μ	Particle
2	2-form $B_{\mu\nu}$	String
3	3-form $C_{\mu\nu\rho}$	Membrane

2-form and 3-form gauge fields parametrize the axionic kinetic terms and scalar potentials

Constraints on kinetic terms and potentials can be translated to properties of strings and membranes

WGC for strings/membranes → underlying Distance and dS conjectures

Dual formulation in terms of 2,3-forms

4D N=1 EFT: $$S = \int \left(\frac{M_P^2}{2} R - M_P^2 K_{\alpha\bar{\beta}}\, d\phi^\alpha \wedge *d\bar{\phi}^{\bar{\beta}} - V \right)$$

$$(s^i, a^i) \;\rightarrow\; (l_i, B_{2i})$$

$$f_a \;\rightarrow\; C_3^a$$

$$-\frac{1}{2} \int G^{ij} \left(M_{\rm P}^2\, d\ell_i \wedge *d\ell_j + \frac{1}{M_{\rm P}^2} H_{3\,i} \wedge *H_{3\,j} \right)$$

$$G_{ij} \equiv \frac{1}{2} \frac{\partial^2 K}{\partial s^i \partial s^j}$$

$$-\int \frac{1}{2} T_{ab} F_a^a * F_4^b$$

$$V = \frac{1}{2} T^{ab} f_a f_b$$

Field metric = 2-form gauge couplings

Potential = 3-form gauge couplings

No force identities

We add BPS charged objects: $-\int d^{p+1}\xi\, T(\phi)\sqrt{-h} + e\int B_{p+1}$

Strings ➡ $T_{\mathbf{e}} = M_{\rm P}^2 |e^i \ell_i|\,, \qquad Q_{\mathbf{e}} = M_{\rm P}\sqrt{G_{ij}e^i e^j}$

Membranes ➡ $T_{\mathbf{q}} = 2M_{\rm P}^3\, e^{\frac{1}{2}K}\, |q_a \Pi^a|\,, \qquad Q_{\mathbf{q}} = M_{\rm P}\sqrt{T^{ab}q_a q_b}$

They satisfy (off-shell):

$$\|\partial T_{\rm str}\|^2 = M_{\rm P}^2 Q_{\mathbf{e}}^2$$

$$\|\partial T_{\rm mem}\|^2 - \frac{3}{2}T_{\rm mem}^2 = M_{\rm P}^2 Q_{\mathbf{q}}^2$$

They look as a no-force condition: (see also [Herraez'20])

$$G^{ij}\partial_i T \partial_j T + \frac{(p+1)(1-p)}{2}T^2 = F^{ab}q_a q_b$$

Interpretation

Low codimension objects → change asymptotic structure of vacuum

How to define T and Q?

Localised operators entering the EFT rather than states of a vacuum

$$-\int d^{p+1}\xi\, T(\phi)\sqrt{-h} + e\int B_{p+1}$$

[Polchinski'14]

Brane couplings should be regarded as defined at the EFT cut-off Λ

Classical back reaction → Classical RG flow $T(\Lambda)$

codim	brane coupling
> 2	irrelevant
2	marginally relevant
1	relevant

Strings

RG flow: $s(r) = s_0 + \frac{e}{2\pi}\log\frac{r}{r_0}$ ➡ $\frac{T(\Lambda)}{M_{\rm P}^2} = \frac{e}{2s_0 + \frac{e}{\pi}\log(\Lambda r_0)}$

At the core of the string $(r \to 0)$: $s(\Lambda \to \infty) \to \infty$, $T(\Lambda \to \infty) \to 0$

Weakly coupled strings ➡ Infinite field distance limits

$$\Delta\phi = \int_{s(r_0)}^{s(r_{max})} \sqrt{G_{ij}ds^i ds^j} = \int_0^{\sigma(r_{max})} Q_{\mathbf{e}}(\sigma)d\sigma \geq \gamma \log\frac{T_{\mathbf{e}}(r_{max})}{T_{\mathbf{e}}(r_0)}$$

$$\sigma = \frac{e}{2\pi}\log(r/r_0)$$

$$Q \geq \gamma\, T$$

Cut-off $\Lambda_{max}^2 \equiv T_{\mathbf{e}}(\Lambda_{max}) < T_{\mathbf{e}}(r_0)\exp\left(-\frac{1}{\gamma}\Delta\phi\right)$

WGC for strings ➡ SDC with $\lambda = 1/\gamma$

Membranes

Classical backreaction: $$T_{\mathbf{q}}^{\text{eff}}(\Lambda) = \frac{T_{\mathbf{q}}}{1 - \frac{kT_{\mathbf{q}}}{2M_{\text{P}}^2\Lambda}}$$

At the asymptotic limits, we can identify some membranes satisfying:

$$K_i = -\frac{d_i}{2is_i} \quad \Rightarrow \quad \partial_\alpha T_{\mathbf{q}} = K_\alpha \sigma_\alpha T_{\mathbf{q}} \quad \Rightarrow \quad T(\Lambda) = \gamma\, Q(\Lambda)\, M_p$$

$$\gamma = \underbrace{2d_1\sigma^2}_{\hat{\alpha}^2/2} - \frac{3}{2}$$

exponential rate of 3-form gauge coupling

These extremal membranes satisfy $\|\partial Q_{\mathbf{q}}^2\| = \hat{\alpha}\, Q_{\mathbf{q}}^2$

If full potential given by: $Q^2 = 2V$ $\Rightarrow$ $\|\partial V\| = cV$ with $c = \hat{\alpha}$

(see also [Grimm,Li, IV'19])

dS conjecture! [Obied et al'18]

Recall that in N=2: $\lambda = \alpha/2$

(exp rate of 1-form gauge coupling)

Could $\frac{|\nabla M|^2}{M^2} \sim \frac{|\nabla V|}{V}$? [Andriot et al'20]

Summary

- Bubbles of nothing are far more common than thought. Topological and dynamical obstruction typically absent in string compactifications.

- The SDC tower also satisfies the WGC at the weak coupling limits, which fixes the exponential mass rate in terms of black hole extremality.

- Swampland conjectures (WGC, SDC, dS conjecture…) get connected by the physics of strings and membranes in 4d N=1 EFT's.

Thank you!