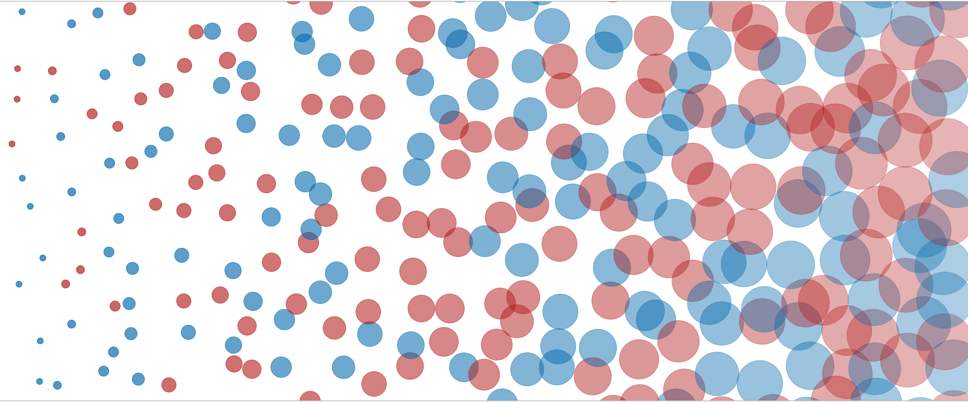
WEAK GRAVITY WITHOUT INSTANTONS



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UNIVERSITY OF AMSTERDAM

String Phenomenology 2020, Friday, June 12

Main Question | What does the axionic Weak Gravity Conjecture mean?

[Arkani-Hamed, Motl, Nicolis and Vafa '06]

What happens physically when we try to restore a global **shift** symmetry?

$$V_{\text{eff}}(\theta) = \sum_{\ell} A_\ell e^{-S_\ell} (1 - \cos \ell\theta) \qquad S_\ell \lesssim |\ell| M_{\text{pl}}/f$$

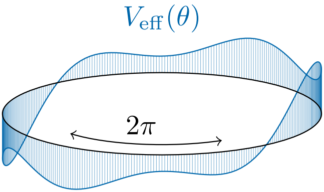
Other WGCs are clear: objects become light for $g \ll 1$ and EFT breaks down:

$$m \lesssim |q| g M_{\text{pl}}$$

But instantons are **not** physical objects, so aWGC is different.

This Talk | There are light **states** that appear for some θ when $S_\ell \ll 1$. Slowly decaying Fourier coefficients **indicate** that a phase transition can occur. Will relate this to a **metric** that diverges at these transitions.

[Banks, Dine, Fox, Gorbatov '03, Svrček, Witten, '06, Rudelius, '15; Brown, Cottrell, Shiu, Soler '15, and many others]



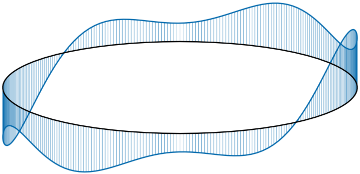
Will think of effective potential as the **ground state energy** of a theory

$$V_{\text{eff}}(\theta) = -\lim_{\beta\to\infty} \frac{1}{\beta} \log \mathcal{Z}(\beta, \theta) = \sum_{\ell \in \mathbb{Z}} v_\ell e^{i\ell\theta}$$

and the axion θ as a **non-dynamical parameter**.

Will also imagine there are other parameters we can tune to vary the v_ℓ.

What controls the topological expansion?



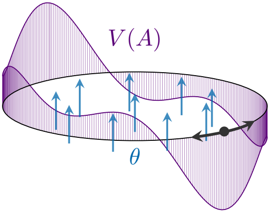
$$V_{\text{eff}}(\theta) = \sum_{\ell \in \mathbb{Z}} v_\ell e^{i\ell\theta}$$

Let's look at a simple **toy model**. Can mimic

$$S_{4\mathrm{d}} = \int \mathrm{d}^4x \left[-\frac{1}{4g^2}F^2 + \frac{\theta}{16\pi^2}F\tilde{F} + \mathcal{L}_{\mathrm{cm}}\right] \qquad A \sim A + \omega$$

with a particle on a circle in an electromagnetic field $(\mathrm{U}(N)$ on $\mathbb{R} \times \mathrm{S}^1)$

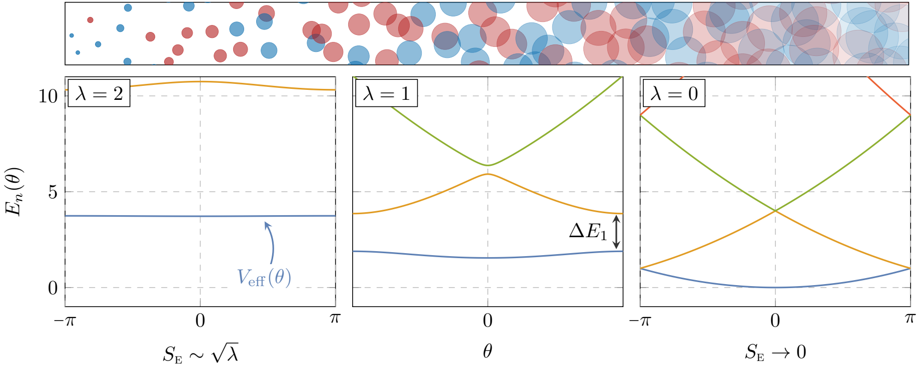
$$S_{1\mathrm{d}} = \int \mathrm{d}t \left[\frac{1}{2}\dot{A}^2 - \theta\dot{A} - V(A)\right] \qquad A \sim A + 1$$



$$\mathcal{H} = \frac{1}{2}(p_A + \theta)^2 + \lambda\left(1 - \cos 2\pi A\right) .$$

[Asorey, Esteve, and Pacheco '83]

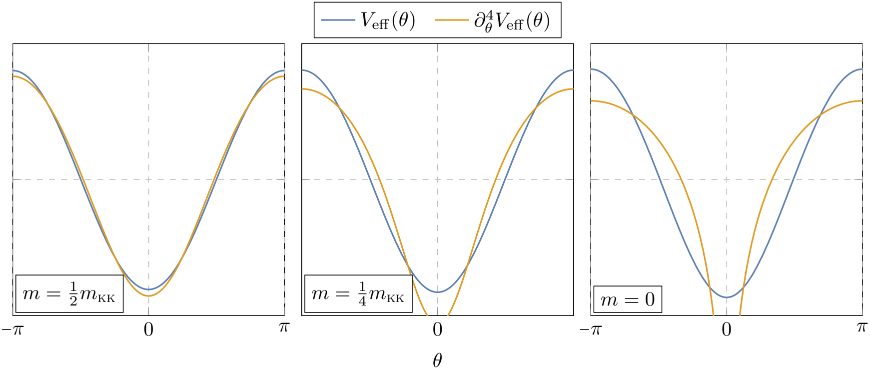
$$\mathcal{H}(\theta) = \frac{1}{2}(p_A + \theta)^2 + \lambda(1 - \cos 2\pi A)$$



A **gap closes** and the effective potential becomes **cuspy** when the topological expansion "fails."

Consider **extranatural inflation** on $\mathbb{R}^{1,d} \times \mathrm{S}^1_R$ with $\Phi(R) = e^{i\theta}\Phi(0)$,

$$S_{\mathrm{E}} = -\sum_{\ell\in\mathbb{Z}} \int \mathrm{d}^{d+1}x \left[|\partial\Phi_\ell|^2 + \left(m^2 + m_{\mathrm{KK}}^2(\theta + 2\pi\ell)^2\right) |\Phi_\ell|^2 \right]$$

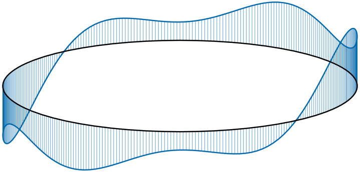


$$v_\ell \sim \exp\left(-2\pi\ell m / m_{\mathrm{KK}}\right)$$

Nice potential, but **discontinuous** d-th derivative as **gap closes**.

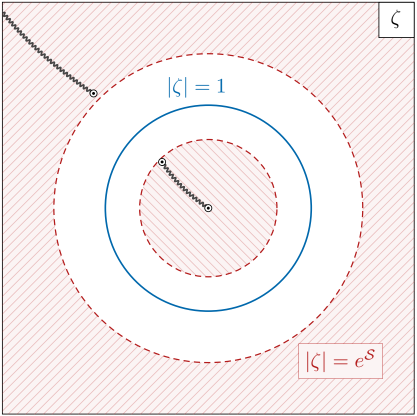
[Hosotani '83; Arkani-Hamed, Cheng, Creminelli, and Randall '03]

What controls the topological expansion?



$$V_{\text{eff}}(\theta) = \sum_{\ell \in \mathbb{Z}} v_\ell e^{i\ell\theta}$$

Helpful to complexify $\zeta = e^{i\theta}$ and consider $V_{\text{eff}}(\zeta) = \sum_{\ell \in \mathbb{Z}} v_\ell \zeta^\ell$.

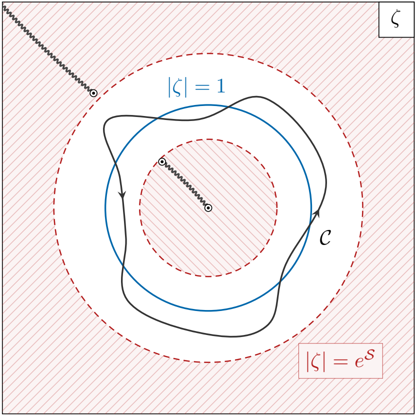


If smooth, then $V_{\text{eff}}(\zeta)$ analytic in annulus about the **physical domain**:

$$e^{-\mathcal{S}} \leq |\zeta| \leq e^{\mathcal{S}}.$$

This annulus is restricted by $V_{\text{eff}}(\zeta)$'s closest **singularities**.

$$v_\ell = \frac{1}{2\pi i}\oint_{\mathcal{C}} \frac{\mathrm{d}\zeta}{\zeta^{\ell+1}} V_{\text{eff}}(\zeta)\,.$$

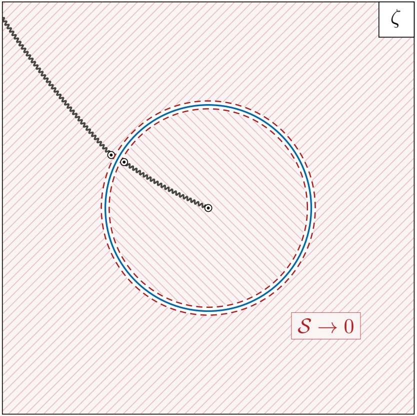


Topological expansion diverges for ζ outside this annulus, so

$$v_\ell \sim e^{-|\ell|\mathcal{S}} \quad \text{as} \quad \ell \to \pm\infty$$

[Boyd '89]

What happens as these singularities impinge upon $|\zeta| = 1$?



They introduce a **discontinuities** in the potential,
nature of singularities dictate the algebraic decay of the v_ℓ:

$$v_\ell \sim \frac{e^{-\ell \mathcal{S} - i\ell a}}{2\pi (i\ell)^{k+1}} \, \text{disc}\, \partial_\theta^k V_{\text{eff}}(a) + \dots \qquad \ell \to \infty$$

Topological expansion is globally sensitive to non-analyticities in $V_{\text{eff}}(\zeta)$!

Main Point | Topological expansion "fails" when the theory experiences a **phase transition** somewhere along the axion's field space.

Should think of the v_ℓ as an **indicator** that a phase transition can occur.

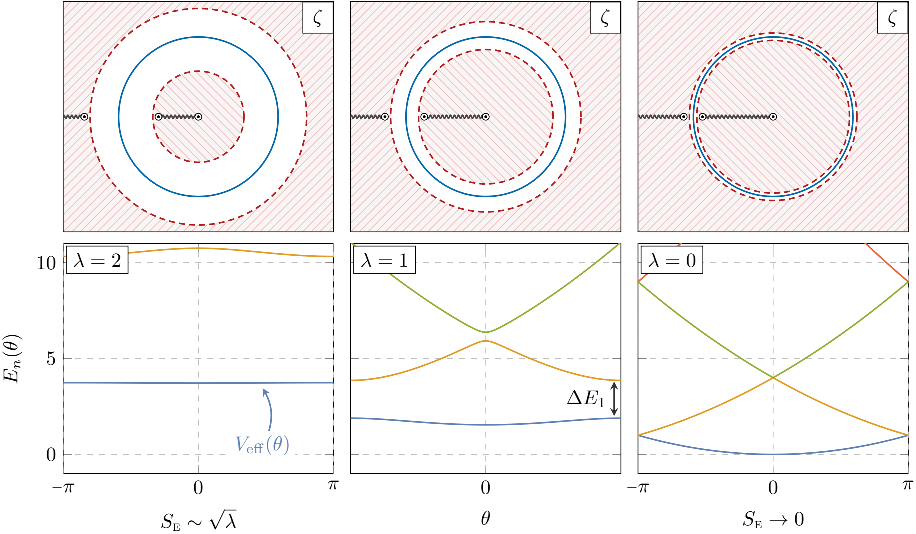
$$\mathcal{Z}(\beta, \zeta) = e^{-\beta E_0(\zeta)}\left[1 + \sum_i e^{-\beta \Delta E_i(\zeta)}\right] \sim \prod_n (1 - \zeta/\zeta_n)$$

Non-analyticities appear at $\mathcal{Z}$'s zeros, poles, etc.

$$V_{\text{eff}}(\zeta) \sim -\sum_n \log\left(1 - \zeta/\zeta_n\right)$$

[Lee and Yang '52, Akemann, Lenaghan, and Splittorff '01, Aguado and Asorey '02]

$$\mathcal{Z}(\beta,\zeta) = e^{-\beta E_0(\zeta)}\left(1 + e^{-\beta \Delta E_1(\zeta)} + \cdots\right)$$



Consider **extranatural inflation** on $\mathbb{R}^{1,d} \times \mathrm{S}^1_R$ with $\Phi(R) = e^{i\theta}\Phi(0)$,

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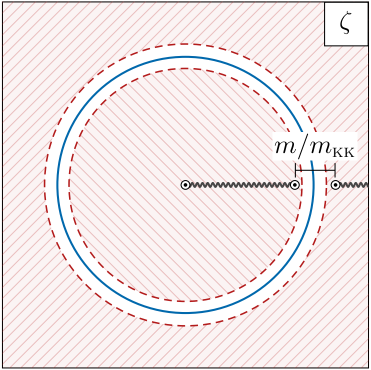
Partition function has **poles** at

$$(2\pi n/\beta)^2 + \mathbf{k}^2 + m^2 + m_{\mathrm{KK}}^2\theta_{n,\mathbf{k}}^2 = 0\,, \qquad n \in \mathbb{Z}$$



Main Point | Topological expansion “fails” when the theory experiences a phase transition somewhere along the axion’s field space.

Should think of v_ℓ as an **indicator** that a phase transition can occur.

Are there other indicators?

Quantum Information Metric

Argued that derivatives of effective potential are sensitive to light states,

$$\frac{1}{2}\frac{\partial^2 V_{\text{eff}}(\lambda)}{\partial\lambda^i\partial\lambda^j} = \sum_{n\neq\Omega} \text{Re}\left[\frac{\langle\Omega(\lambda)|\partial_i\mathcal{H}|n\rangle\langle n|\partial_j\mathcal{H}|\Omega(\lambda)\rangle}{V_{\text{eff}}(\lambda)-E_n(\lambda)}\right]$$

singularities mess with convergence of topological expansion.

$$\mathcal{H}(\lambda)|\Omega(\lambda)\rangle = V_{\text{eff}}(\lambda)|\Omega(\lambda)\rangle$$

Can also consider the **information metric** or **fidelity susceptibility**

$$g_{ij}(\lambda) = \sum_{n\neq\Omega} \text{Re}\left[\frac{\langle\Omega(\lambda)|\partial_i\mathcal{H}|n\rangle\langle n|\partial_j\mathcal{H}|\Omega(\lambda)\rangle}{(V_{\text{eff}}(\lambda)-E_n(\lambda))^2}\right].$$

This is another **indicator** of a phase transition.

This **information metric** is proportional to the **Zamolodchikov** metric

$$g_{ij}(\lambda) \propto G^{\text{z}}_{ij}(\lambda)$$

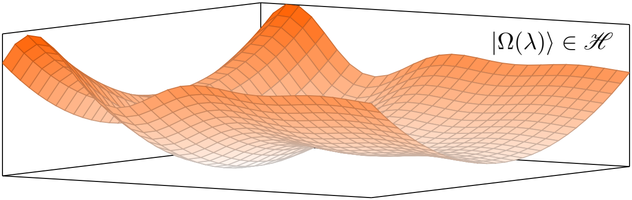
if λ^i parameterize the moduli space of a CFT, e.g. the Weil-Petersson metric.
[Candelas, Hübsch, and Schimmrigk '90; Cvetič, Ovrut, and Louis '89]

Can be generalized to non-marginal deformations.

In local theories, a divergence in g_{ij} **indicates** there is a gapless mode.
[Venuti and Zanardi, '07; Zhou and Barjaktarevič, '07; Gu '09]

[see also Balasubramanian, Heckman, and Maloney '14; Miyaji, Numasawa, Shiba, Takayanagi, and Watanabe '15]

This is also the **Fubini-Study** metric on vacuum manifold $|\Omega(\lambda)\rangle$.

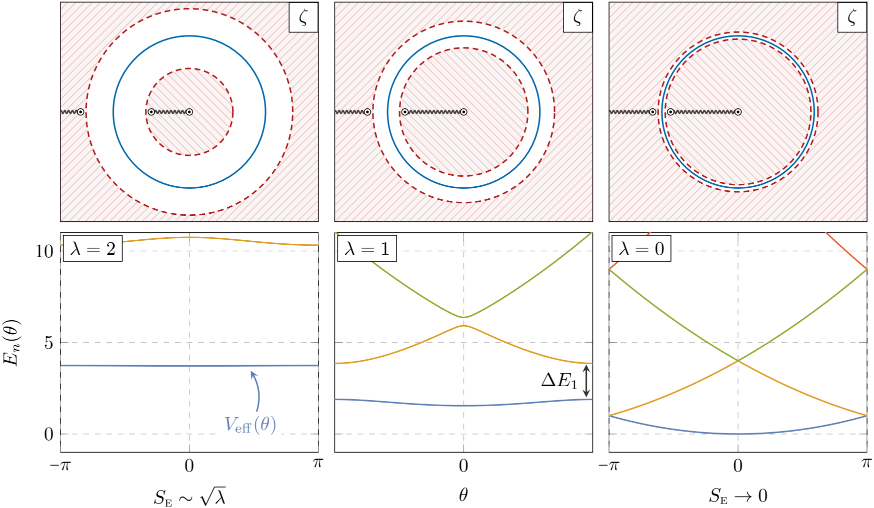


Operational definition | Notion of distance based on statistical distinguishability. How well can I distinguish one state (or theory) from another using the most discerning measurement?

Two states are infinite distance in this metric if a **single measurement** (like whether or not a charge is conserved) can distinguish them in principle.

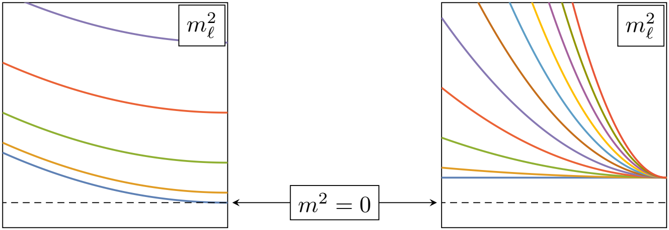
[Wootters, '81; Provost and Vallee, '80]

Finite distance



$$g_{\theta\theta} \propto \frac{\lambda^2}{[(\pi-\theta)^2+\lambda^2]^2}$$

$$S_{\mathrm{E}} = -\sum_{\ell} \int \mathrm{d}^{d+1}x \left[|\partial \Phi_\ell|^2 + \left(m^2(\lambda^a) + m_{\mathrm{KK}}^2(\lambda^i)\ell^2 \right) |\Phi_\ell|^2 \right]$$



$$g_{ab} \propto \frac{\partial_a m^2 \partial_b m^2}{m^{4-d}}$$

$$g_{ij} \propto \frac{\partial_i m_{\mathrm{KK}}^2 \, \partial_j m_{\mathrm{KK}}^2}{m_{\mathrm{KK}}^4}$$

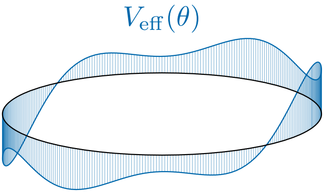
Finite distance

Algebraic decay

Infinite distance

Exponential decay

Conclusions



Main question | What is the axionic Weak Gravity Conjecture telling us?

- Should interpret as a statement about **states becoming light.**
- When the topological expansion "fails," a **phase transition** must occur **somewhere** along the axion's field space.
- Nature of this transition encoded in asymptotics of harmonics.
- Can also phrase this as the divergence of the **information metric**.

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