THE TADPOLE PROBLEM

2x [200X.XXXX] with I. Bena, J. Blåbäck, M. Graña
[1809.06861], [1910.08094], [1912.09948] with I. Bena, A. Buchel, E. Dudas, M. Graña

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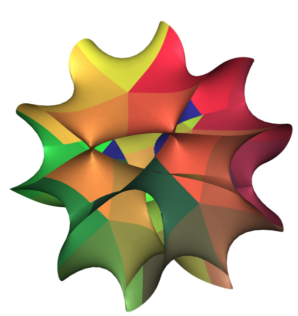
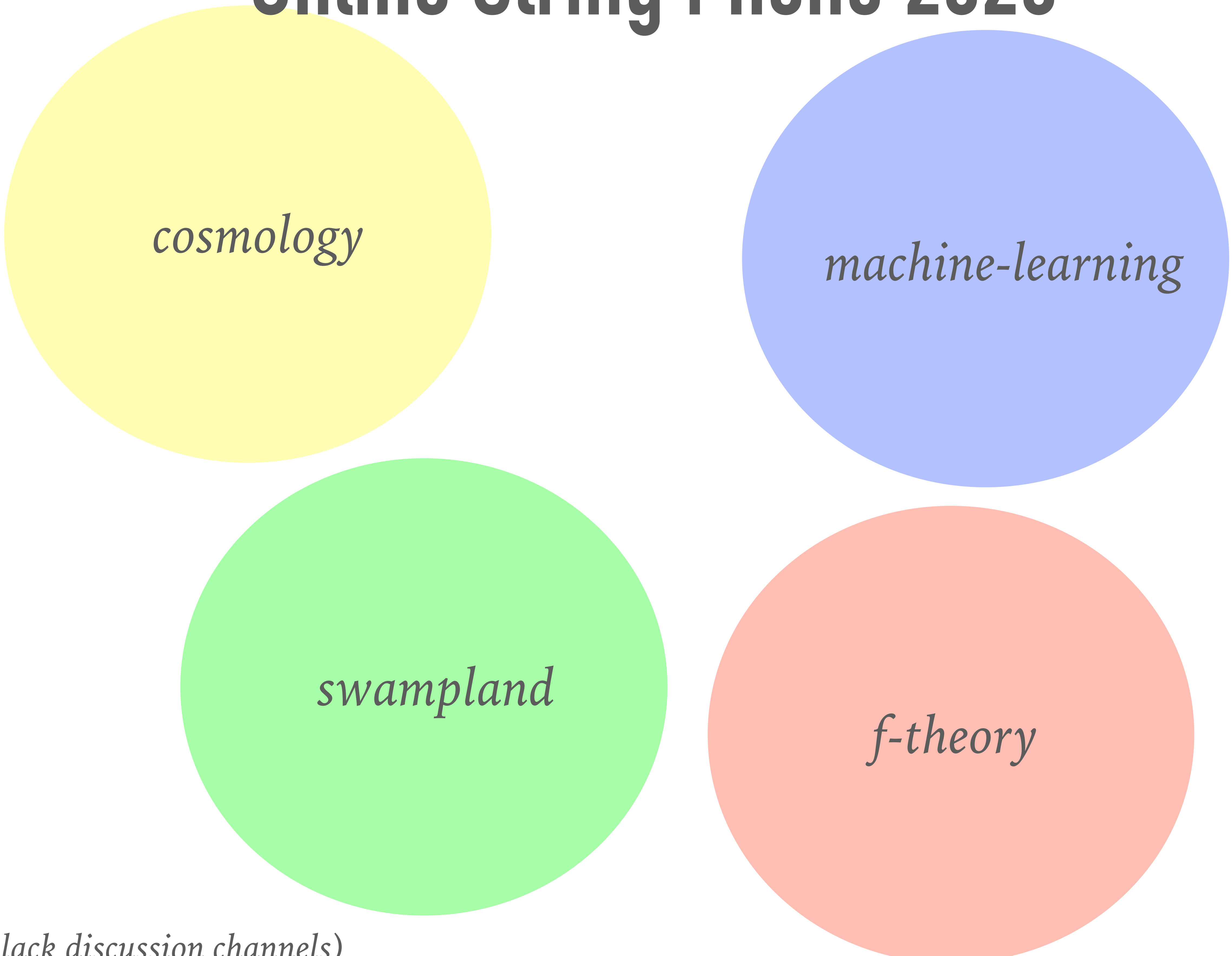


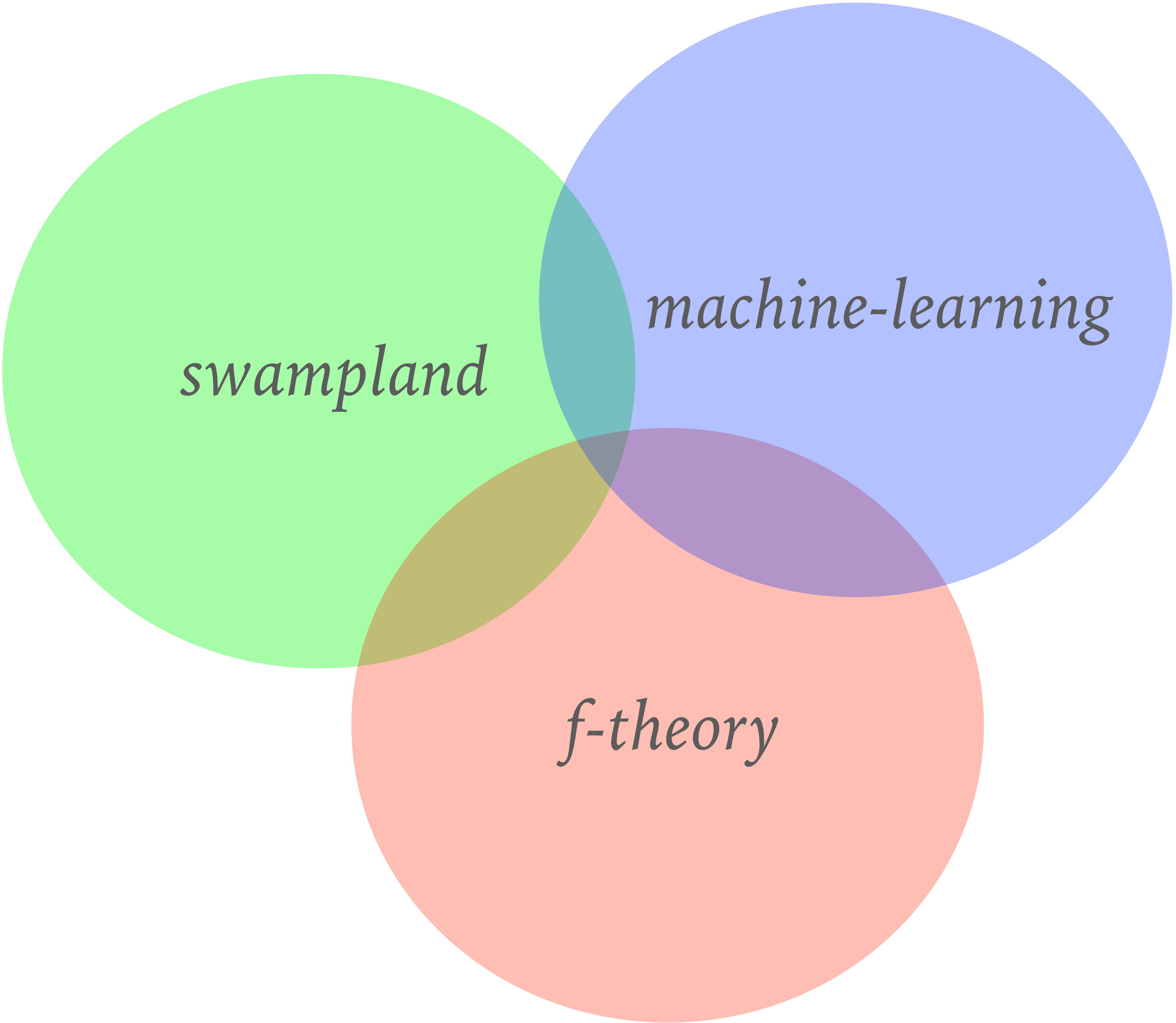
String Pheno 2020, June 12

Online String Pheno 2020



(slack discussion channels)

The Tadpole Problem



MOTIVATION

- Flux compactifications:
  Huge landscape of vacua: $10^{(\text{big number})}$
  [Ashok, Douglas '03], [Denef, Douglas '04], [Taylor, Wang '15]
- CYs with large Hodge numbers: many choices for fluxes
- Constraints on fluxes:
  - Integer quantization
  - Tadpole cancelation
- Goal/difficulty: consistent vacuum + stabilize all moduli within these constraints! *(see also A. Cole's, J. Moritz' and many others' talks)*
- Here: Issues at large numbers of moduli?

TADPOLE CANCELATION

- In IIB:

$$\int F_3 \wedge H_3 + Q_3^{loc} = 0$$

- D3-charge of localized sources:

$$Q_3^{\text{loc}} = N_{D_3} - N_{O3} - \frac{1}{24}(\chi(D7) + 4\chi(O7)) - \frac{1}{2}\int_S \left(\mathrm{tr}(\mathrm{F} \wedge \mathrm{F}) - \mathrm{tr}\,\mathrm{F} \wedge \mathrm{tr}\,\mathrm{F}\right)$$

- F-theory:

$$\frac{1}{2}\int G_4 \wedge G_4 = \frac{\chi(CY_4)}{24}$$

$\chi(CY_4)$: Euler number of CY 4-fold

(encodes IIB D7-charges)

THE TADPOLE CONJECTURE

[Bena, Blåbäck, Graña, SL, to appear]

- Euler number in terms of Hodge numbers:

$$\chi(CY_4) = 6(8 + h^{1,1} + h^{3,1} - h^{2,1})$$

#moduli to be stabilized by G_4

- Scaling with $h^{3,1}$:

$$\frac{\chi(CY_4)}{24} \propto \frac{1}{4} h^{3,1}$$

(here $h^{1,1}$ small)

Tadpole conjecture: (large $h^{3,1}$) $\quad \frac{1}{2}\int G_4 \wedge G_4 \Big|_{\text{all mod. stab.}} \gtrsim \alpha h^{3,1}$

$\rightarrow$ no large $h^{3,1}$ vacua if $\alpha > \frac{1}{4}$

Strong version: $\alpha = \frac{1}{4}$?

PLAN FOR THE REST OF THE TALK

1. Application / Implications

   de Sitter vacua from $\overline{D3}$-branes
   in the Klebanov-Strassler throat (KKLT)

2. Examples

   a) D7-branes in IIB
   [Collinucci, Denef, Esole '08]

   b) M-theory on K3 x K3
   (using an evolutionary algorithm)

APPLICATION:

UPLIFTING RUNAWAYS

FLUXES & COMPLEX STRUCTURE OF DEFORMED CONIFOLD

- KKLT: $\overline{D3}$-branes in warped deformed conifold (Klebanov-Strassler)

deformed conifold:

$$\sum_{i=1}^{4} z_i = S$$

warp factor:

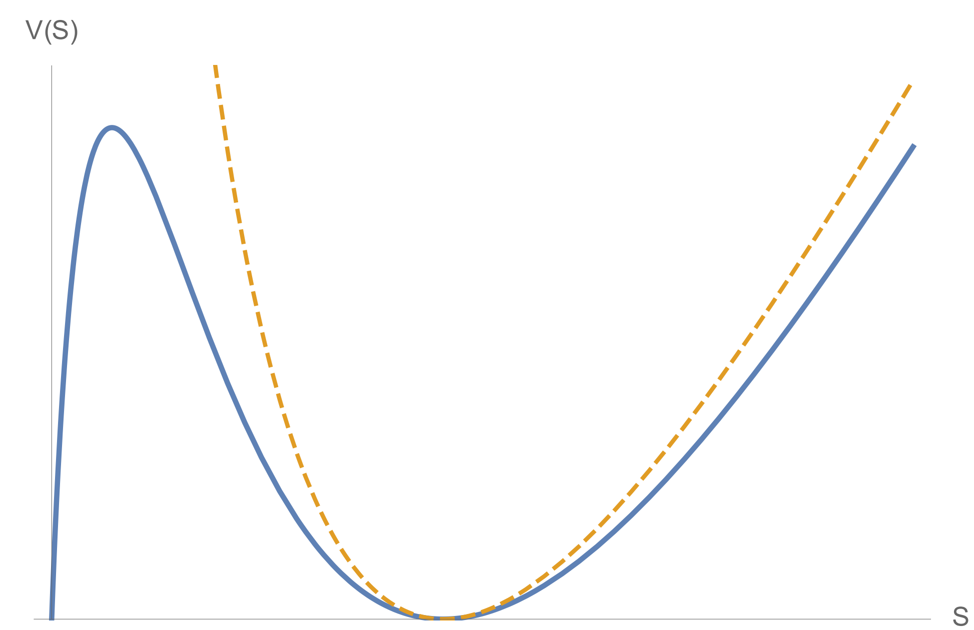
$$e^{-4A(\tau)} = \frac{g_s(\alpha' M)^2}{|S|^{4/3}} I(\tau)$$

$\rightarrow$ *S: complex structure modulus, controls hierarchy*

- Flux potential for S:

$$V_{KS} \sim \frac{|S|^{4/3}}{g_s(\alpha' M)^2} \left| \frac{M}{2\pi i} \log \frac{\Lambda_0^3}{S} + i \frac{K}{g_s} \right|^2$$

[Douglas, Shelton, Torroba, '07, '08]



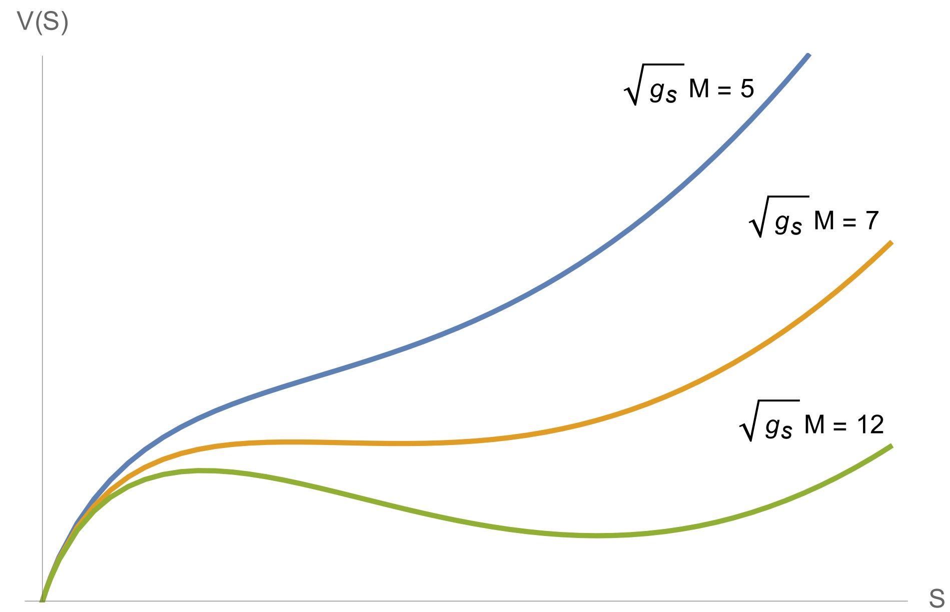
UPLIFTING RUNAWAYS

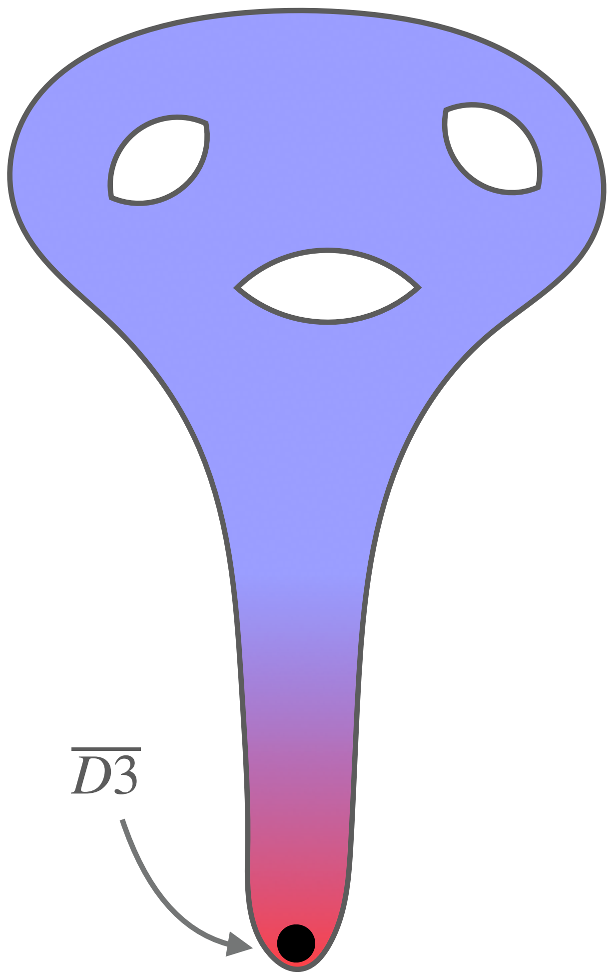
[Bena, Dudas, Graña, SL '18]

➤ Potential for S from $\overline{D3}$:

$$V_{\overline{D3}} \sim 2e^{4A}T_{D3} \sim \frac{|S|^{4/3}}{g_s(\alpha' M)^2}$$

➤ $V_{KS} + V_{\overline{D3}}$:





➤ Runaway to singular KT-throat ($S = 0 \to \infty$-redshift) unless:

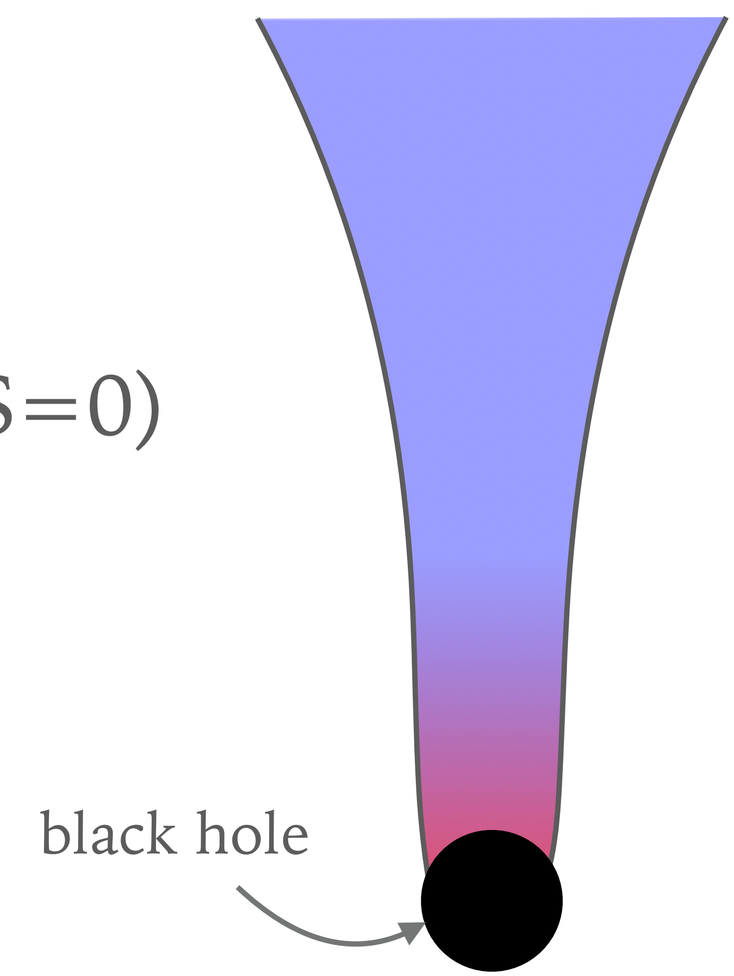
$$\sqrt{g_s}M > \gamma_{\overline{D3}}\sqrt{N_{\overline{D3}}}\,, \qquad \gamma_{\overline{D3}} \approx 6.8$$

KS-BLACK HOLES AND χ-SYMMETRY BREAKING

[Bena, Buchel, SL '19]

- [Buchel '18]: Numerical construction of KS-black hole
- Exists only if

$$\mathcal{E} < \mathcal{E}_{\chi SB}$$



- Higher temp./energy: Only KT-black hole (S=0)
- Express energy-density $\mathcal{E}$ in $\overline{D3}$-units:

$$\mathcal{E} = N_{\overline{D3}} V_{\overline{D3}}$$

- Bound on $\mathcal{E}$ becomes:

$$\sqrt{g_s} M > \gamma_{BH} \sqrt{N_{\overline{D3}}} \, , \qquad \gamma_{BH} \approx 4.2$$

THE RADION OF KKKLT

[Randall '19]

- Effective 5D description:

  KS-modulus $S^{1/3}$

  $\leftrightarrow$

  radion ϕ (length of throat)

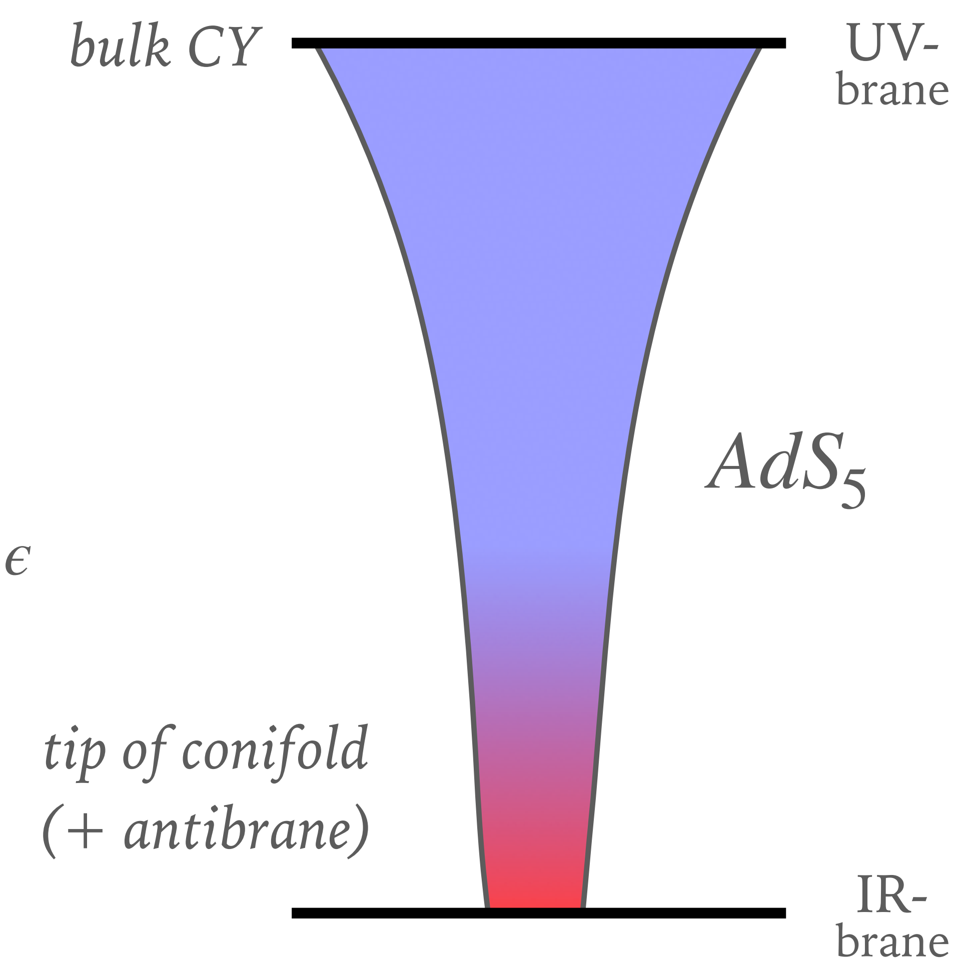
- Goldberger-Wise mechanism:
  $\rightarrow$ potential for ϕ: $V(\phi) = \lambda_1 \phi^4 + \lambda_2 \phi^{4-\epsilon}$

- Potential for S:

$$V(S) = S^{4/3}\left[\kappa_1 - \kappa_2 \log \frac{\Lambda_0^3}{S}\right]^2 \approx \kappa_1^2 S^{4/3}\left[5 - 6\left(\frac{S}{\Lambda_0^3}\right)^{\epsilon}\right]$$

- Antibrane: $\delta V = \delta S^{4/3} \quad \rightarrow \quad$ perturbation of IR brane tension

RS-scenario:



CONSTRAINTS ON HIERARCHY

- Hierarchy in KS: $\frac{\Lambda_{IR}}{\Lambda_{UV}} \sim |S|^{1/3} \sim \exp\left(\frac{2\pi K}{3g_s M}\right)$

uplifting runaway:

$$g_s M^2 > M_{min}^2$$

tadpole cancelation:

$$KM \leq Q_{eff}$$

$$Q_{eff} = Q_{loc} - Q(\text{"other fluxes"})$$

- Constraint on hierarchy (one $\overline{D3}$):

$$h = \frac{2\pi}{3}\frac{MK}{g_s M^2} < \frac{2\pi}{3}\frac{Q_{eff}}{M_{min}^2} \approx 0.045 \times Q_{eff}$$

- F-theory + tadpole conjecture:

$$Q_{eff} = \frac{\chi(CY_4)}{24} - \frac{1}{2}\int G_4 \wedge G_4 \Big|_{\text{bulk}} \lesssim \left(\frac{1}{4} - \alpha\right) h^{3,1}_{\text{bulk}}$$

strong tadpole conjecture ($\alpha \geq 1/4$) = no large hierarchy

FIRST EXAMPLE:

IIB WITH D7-BRANES

D7-BRANES IN THE SEN-LIMIT

[Collinucci, Denef, Esole '08]

- CY_4: Z elliptically fibered over $\mathbb{CP}^3$

$$Z\colon\ y^2 = x^3 + f(u)xz^4 + g(u)z^6$$

- topological data:

$$\chi(Z) = 24 \times 972\,, \qquad h^{3,1}(Z) = 3878$$

- Sen-limit:

$$f = -3h^2 + \epsilon\eta\,, \qquad g = -2h^3 + \epsilon h\eta - \epsilon^2\chi/12$$

- D7-branes: $\eta(u)^2 = h(u)\chi(u)$

- #D7-moduli: $N_{D7} = \binom{16+3}{3} + \binom{24+3}{3} - \binom{8+3}{3} - 1 = 3728$

$\rightarrow$ Most moduli: D7-brane moduli!

D7-BRANES IN IIB

[Collinucci, Denef, Esole '08]

- D7-branes on degree 2m surface S: $P_{2m} = \eta^2 - h\chi = 0$

$$N_{D7} = \frac{4}{3}m^3 - 8m^2 + \frac{59}{3}m - 1$$

$$(\text{D7-tadpole} \ \rightarrow \ m = 16 \ \rightarrow \ N_{D7} = 3728)$$

- D7-world volume fluxes, dual to degree d curve γ:

$$N_{constr.} = 2md + 1 \quad \Rightarrow \quad d \geq \frac{2}{3}m^2 - 4m + \frac{59}{6}$$

- D3 charge from fluxes:

$$Q_{D3}(\gamma) = \chi(\gamma) + (2m-4)d \geq \frac{4}{3}m^3 - \frac{32}{3}m^2 + \frac{107}{3}m - \frac{118}{3}$$

$$(m = 16 \ \rightarrow \ Q_{D3} > 3276 > 1944 = Q_{loc})$$

$\rightarrow$ weekly coupled IIB: no D7-brane moduli stabilization?!

SECOND EXAMPLE:

M-THEORY ON K3 X K3

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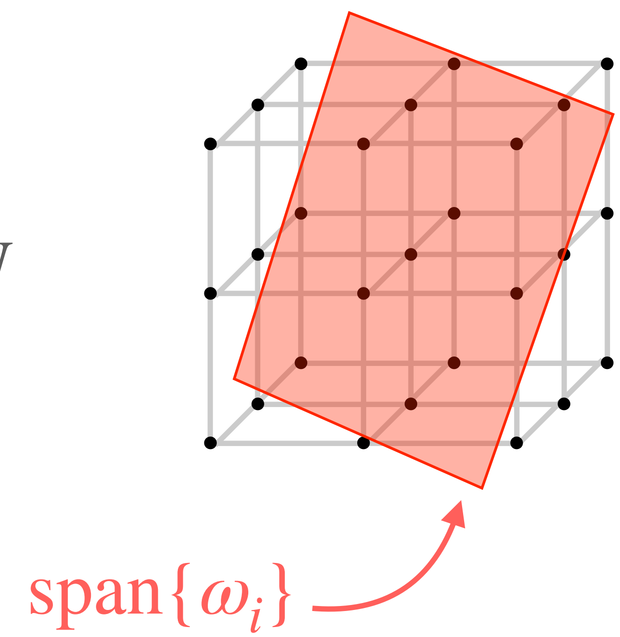
- [Aspinwall, Kallosh '05]:
  Stabilize all complex structure moduli within the tadpole bound:

$$\frac{\chi(K3 \times K3)}{24} = 24$$

- [Braun et al. '08]:
  - Stabilize all moduli (Kähler + complex str.) by fluxes
  - No knowledge of period maps necessary!
- Here:
  - Stabilize all moduli
  - at generic point in moduli space (no singularity)
  - bound on tadpole from fluxes?

THE MODULI SPACE OF K3



- Middle cohomology of K3:

$$H^2(K3,\mathbb{Z}) \cong (-E_8) \oplus (-E_8) \oplus U \oplus U \oplus U$$

even, self-dual lattice of sign. (3,19)

- Point in moduli space:

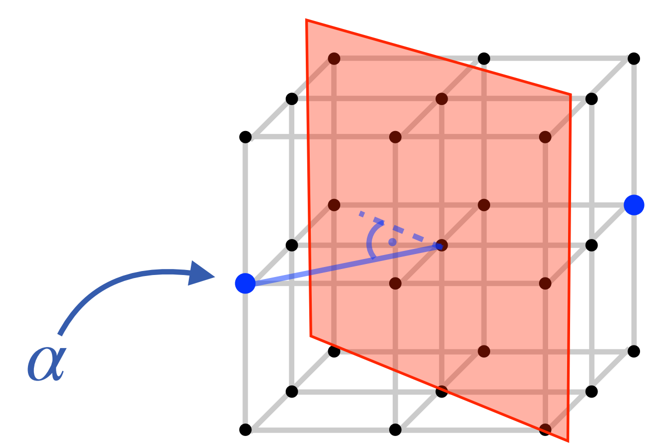
choice of *three* self-dual 3-forms $\omega_i \in H^2(K3,\mathbb{R})$, $\quad i = 1,2,3$

$$\rightarrow \quad \Omega = \omega_1 + i\omega_2 \,, \; J \sim \omega_3$$

- Orbifold singularity:

root $\alpha \in H^2(K3,\mathbb{Z})$ $(\|\alpha\|^2 = -2)$

s.t. $(\alpha, \omega_i) = 0 \quad \forall i$



MODULI STABILIZATION ON K3 X K3

[Braun, Hebecker, Ludeling, Valandro '08]

- 4-form flux: $G_4 \in H^2(K3,\mathbb{Z}) \times H^2(K3', \mathbb{Z})$

$$\rightarrow 22 \times 22 \text{ dim. integer matrix } G^{IJ}$$

- G_4 defines a map $N\colon\ H^2(K3) \rightarrow H^2(K3)$

$$N^I{}_J = G^{IK} d_{KL} G^{LM} d_{MJ} \qquad (d_{IJ}\text{: intersection matrix})$$

- The eigenvectors v_i of N uniquely split into

$$(3 \times v_i^+, 19 \times v_j^-) \ \text{ with } \|v_i^+\|^2 > 0,\ \ \|v_j^-\|^2 < 0$$

if:
- $N^I{}_J$ is diagonalizable with non-neg. eigenvalues
- degenerate eigenvalues have eigenspaces of definite signature

$\rightarrow$ *Minkowski vacuum with all moduli stabilized* $(\omega_i = v_i^+)$

MODULI STABILIZATION ON K3 X K3

- Tadpole:

$$\frac{1}{2}\int G_4 \wedge G_4 = \frac{1}{2}\mathrm{tr}(N)$$

- Goal: Find $G^{IJ} \in \mathbb{Z}^{22\times 22}$:
  - $N^I{}_J$ has desired diagonalization properties $\rightarrow$ *vacuum + mod. stab.*
  - $\nexists$ root $\alpha \perp (\omega_i = v_i^+) \rightarrow$ *smooth K3s („generic point in mod. space")*
  - $\mathrm{tr}(N)$ is minimal $\rightarrow$ *minimal tadpole*

$\rightarrow$ perfect for computer search with

evolutionary algorithm

ML & the Landscape: [He '17], [Ruehle '17], [Carifio, Halverson, Krioukov, Nelson '17], …
genetic algorithms for flux vacua: [Cole, Schachner, Shiu '19]

EVOLUTIONARY ALGORITHM / DIFFERENTIAL EVOLUTION

- Optimization inspired by biological evolution („reproduction“, „mutation“, „selection“, …)
- Goal: Optimize a fitness function $f \colon \mathbb{R}^n \to \mathbb{R}$
- Differential evolution:
  - Random initial population: $P \in \mathbb{R}^{n \times N}$ (N = size of pop.)
  - For each $x \in P$:
    - Mutation: Randomly select $y, z \in P$: $x' = x + \gamma(y - z)$
    - Crossover: x'' = random combination of x and x'
    - Selection: If $f(x'') < f(x)$: Replace x with x''
  - *Repeat …*

DIFFERENTIAL EVOLUTION FOR K3 X K3 [Bena, Blåbäck, Graña, SL, to appear]

- Implemented fitness function for our problem in **Julia** using **BlackBoxOptim.jl** [Feldt et al.] and **bbsearch.jl** [Blåbäck].

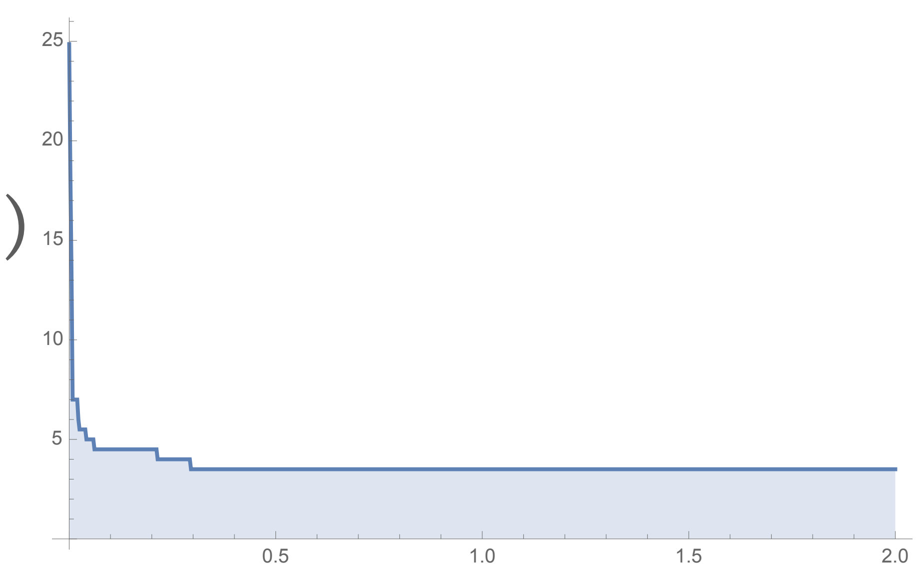
- Challenges:
  - HUGE search space!
    (e.g. $\#(\text{matrices with entries in } \{0, \pm 1\}) \approx 10^{231}$)
  - Finding roots $\perp \text{span}(\omega_i)$ (lattice vectors of minimal length) is NP!

- Slow convergence (multiple weeks…)

EXTRAPOLATING FROM SMALLER LATTICES

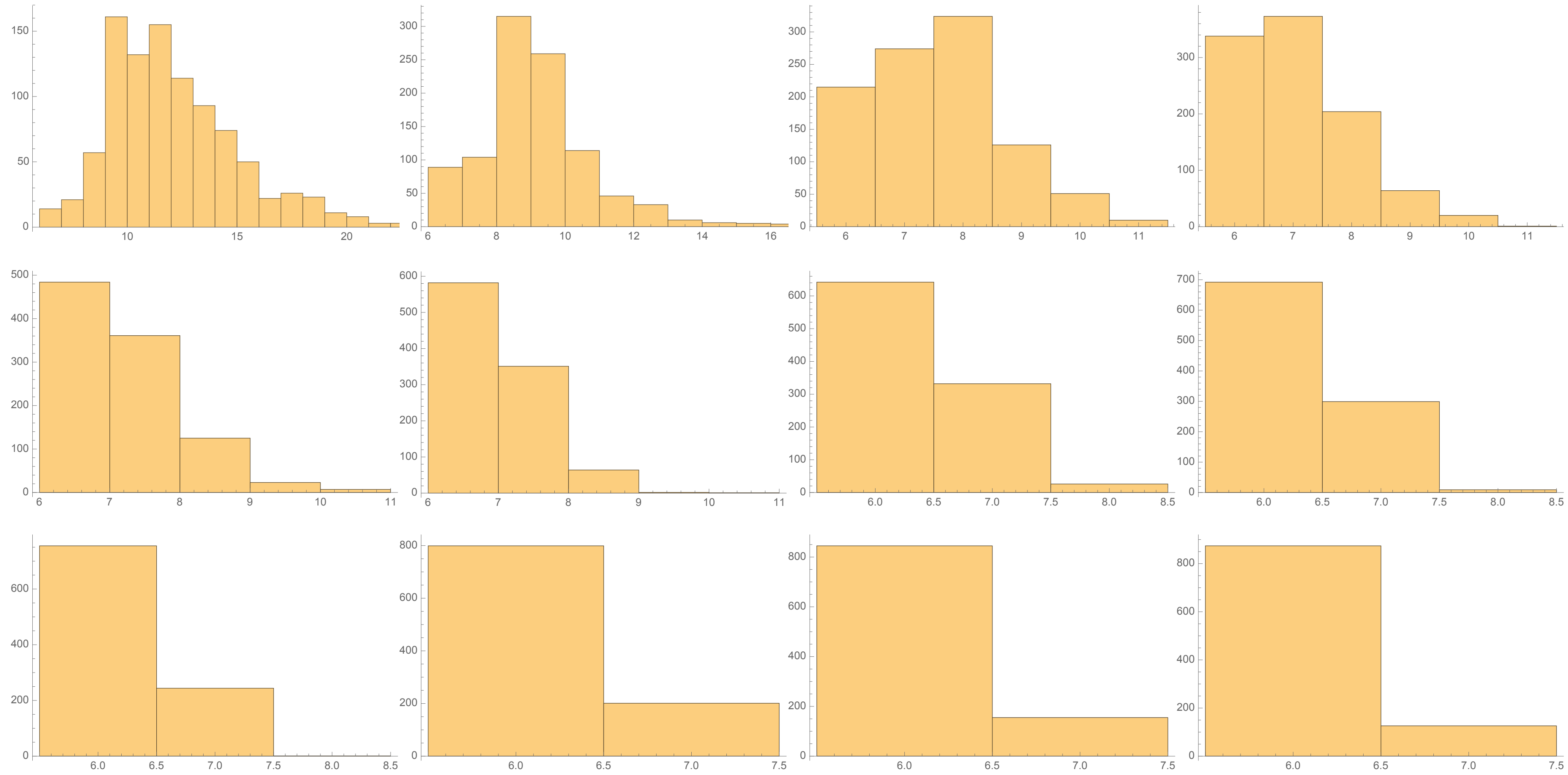
- After *1-2 weeks* runtime: tadpole $t = 24$ found
  (still some possible issues with roots $\perp$ span(ω_i) …)
- Problems:
  - Even longer runtime $\rightarrow$ smaller t possible?
  - Decide when/if minimal t is found?

- Different strategy: Lattices of smaller dimension d
  (e.g. $A_n \oplus U \oplus U \oplus U$, $E_n \oplus U \oplus U \oplus U$, …)
- Result:
  - Fast convergence to $t = d$ (sometimes $t = d - 1$)
  - No lower tadpoles even after long searches
  - Conclusion: $t_{min} \approx d$!



EXAMPLE: 6-DIM. LATTICE

- 6-dim. lattice $U \oplus U \oplus U$
- Time evolution of the tadpoles in a population with N=1000 (snapshot every 120s):



- After 24 min: Almost all matrices arrived at tadpole t=6.

CONCLUSION

Extrapolate to K3-lattice:

- M-theory on K3 x K3:
  - stabilization of all moduli
  - generic point in moduli space (no orbifold singularity)
  - fluxes with arbitrary small M2-charge ($Q \lesssim 22$)

$\rightarrow$ cannot have all three!

fluxes with small charge	$\leftrightarrow$	*additional light d.o.f*

THANK YOU!