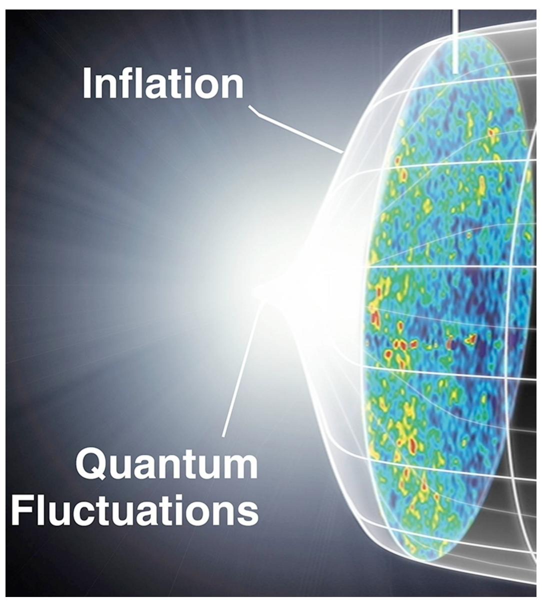
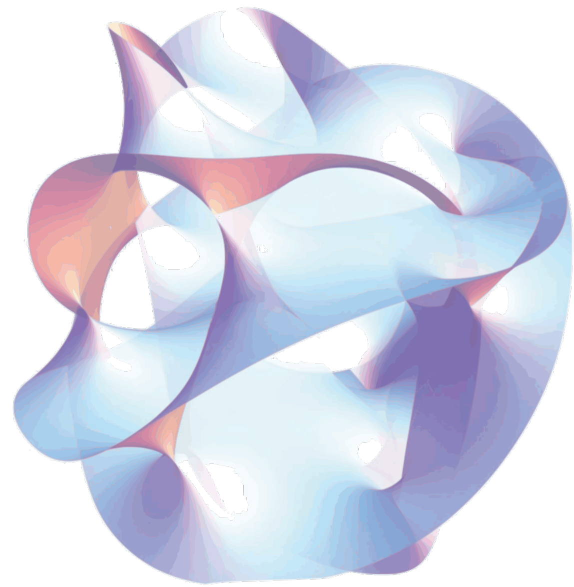
Conformal Bootstrap and Quantum Field Theory

Modern Trends in Particle Physics
NathFest 2019

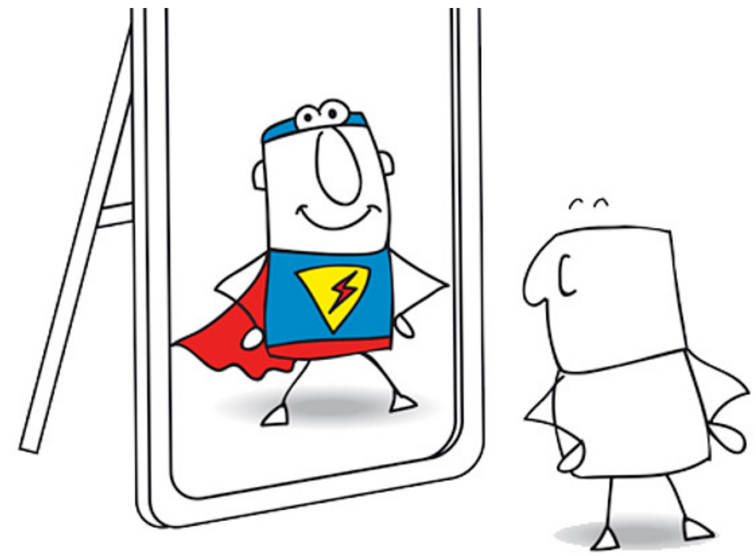
Modern Trends in Particle Physics

inflation



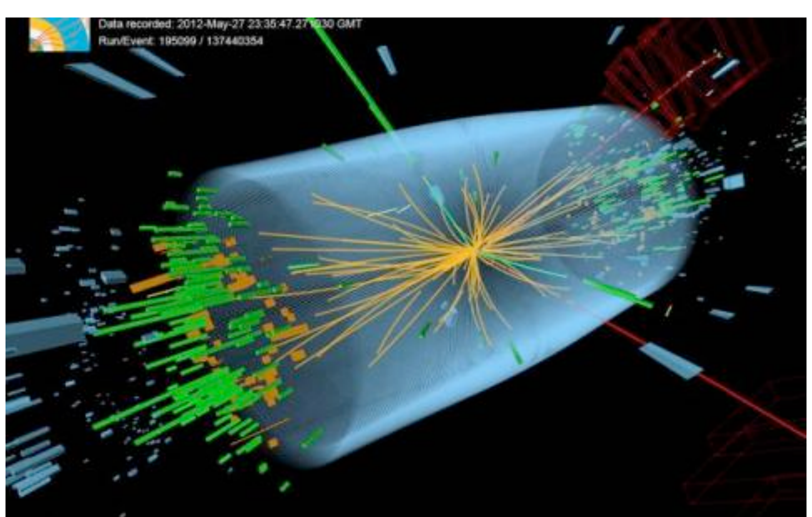


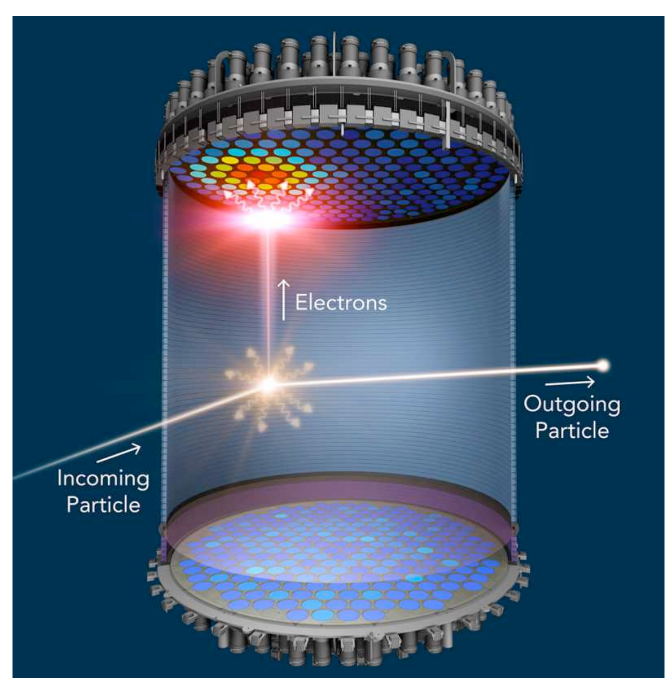
supersymmetry



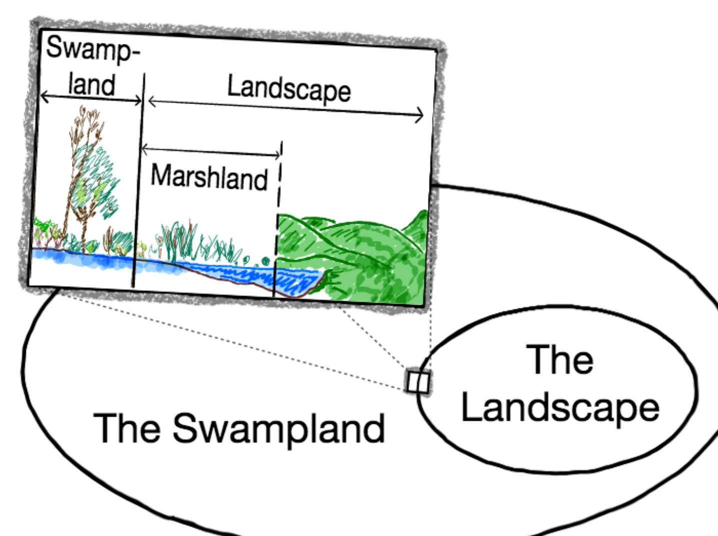
dark matter

colliders





landscape



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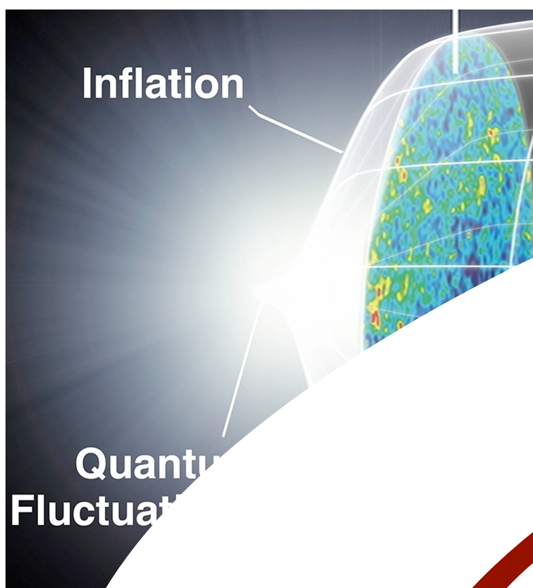
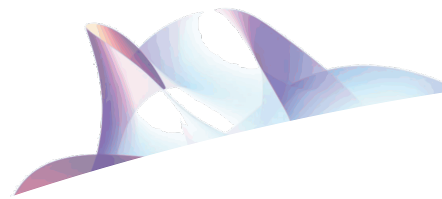


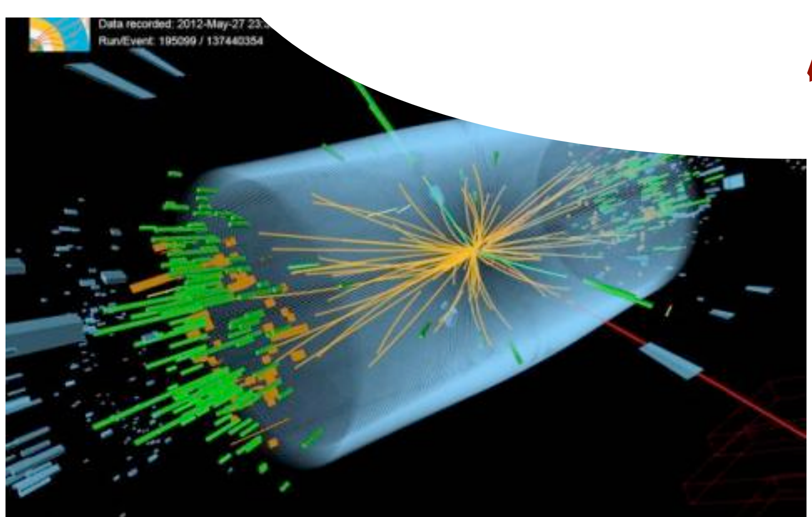
supersymmetry

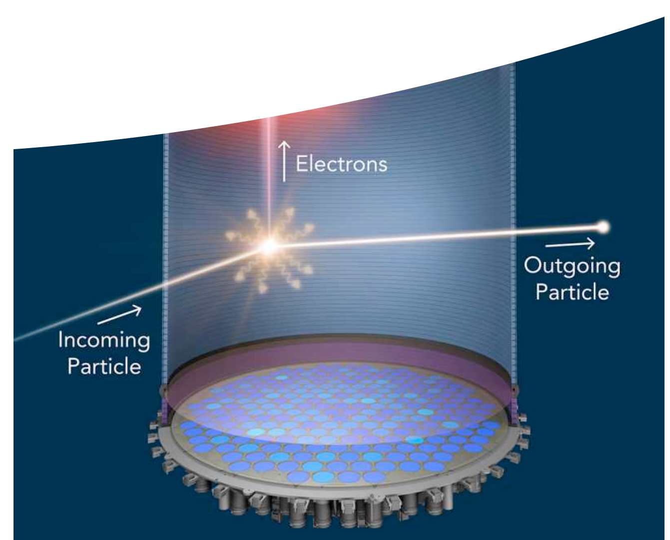


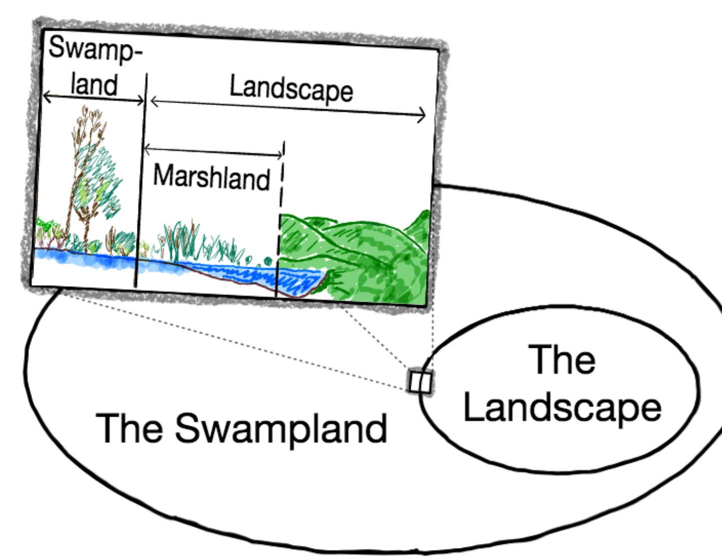
It's all just

QFT

andscape







It’s all just

QFT = CFT + EFT

Philosophy Master: Because there is no other way to express oneself but through prose or poetry. Whatever is not prose, is poetry, and whatever is not poetry is prose.

Jourdain: When I say, Nicole! Bring me my slippers, is that prose?

Philosophy Master: Yes, sir.

Jourdain: So I have been speaking prose for years without even knowing it!

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What is Quantum Field Theory?

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There are a number of different nonperturbative answers

1) Lattice Path Integral

2) S-Matrix

3) Conformal Bootstrap

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1) Lattice Path Integral

QFT is given by Euclidean path integral

Advantages

Disadvantages

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Advantages	Disadvantages
Well-developed tool, gives concrete numeric results for many physical quantities at strong coupling	
Gives a rigorous definition of many QFTs	

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QFT is given by Euclidean path integral

Advantages

Well-developed tool, gives concrete numeric results for many physical quantities at strong coupling

Gives a rigorous definition of many QFTs

Disadvantages

Difficult/impossible to get many Lorentzian quantities

Latticization not always possible (e.g. non-Lagrangian theories, or chiral gauge theories, esp. **Standard Model**!)

The real world is not a lattice

What is Quantum Field Theory?

2) S-Matrix

QFT is given by S-matrix satisfying various consistency conditions (crossing, "analyticity", unitarity)

Advantages	Disadvantages

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Many beautiful and useful patterns emerge	
“Explains” why QFT is necessary (Weinberg)	

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Advantages

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Many beautiful and useful patterns emerge

“Explains” why QFT is necessary (Weinberg)

Disadvantages

Difficult/impossible to construct exact S-matrix beyond perturbation theory

Not all theories have particles and S-matrices

More to life than scattering

What is Quantum Field Theory?

3) Conformal Bootstrap

CFTs=solutions to Bootstrap Equation

QFT is given by RG flows between CFTs

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CFTs are nonperturbatively defined in terms of CFT “data” and bootstrap equation

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Disadvantages

Not always possible to find CFT data in practice

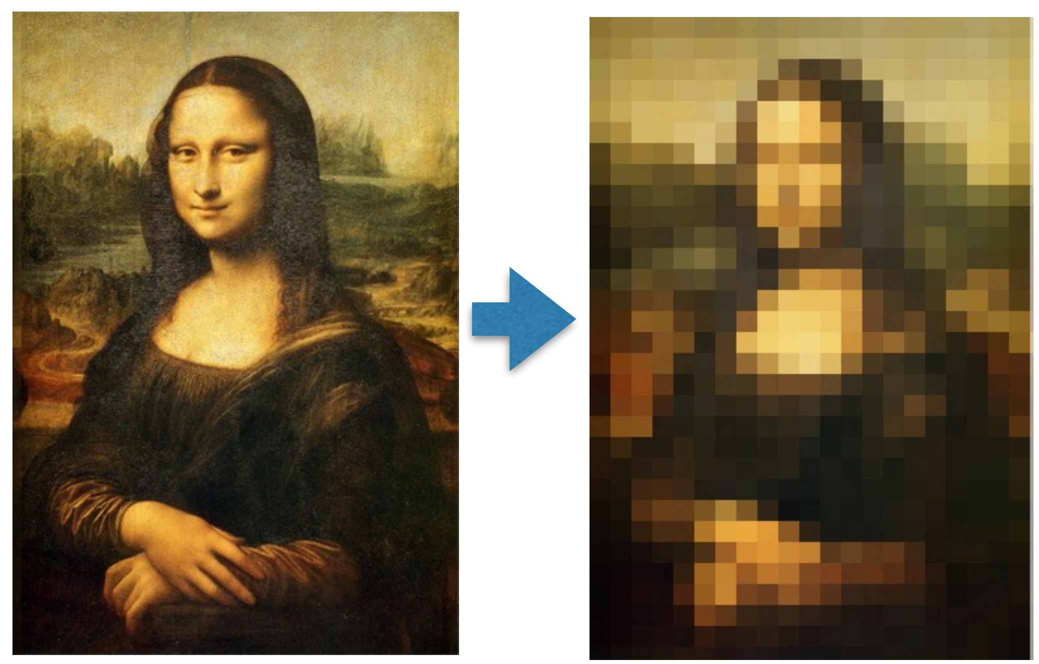
Not all QFTs are CFTs in UV
Sometimes all we have is EFT
e.g. gravity, string theory*

*But string theory = CFT on world sheet,
and gravity in AdS=CFT,
so still just CFT in the UV?

RG Flow

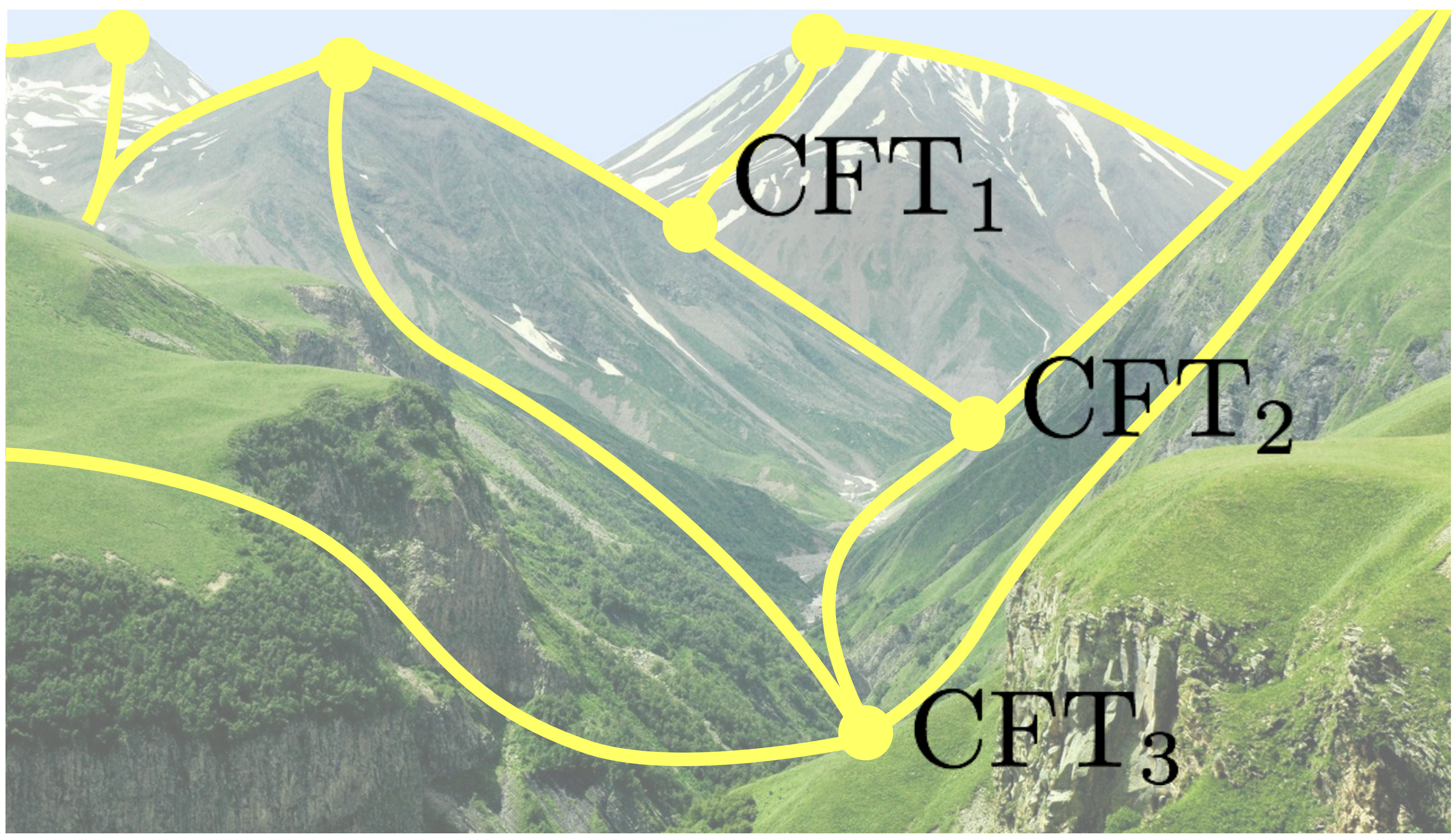
Ken Wilson :

Organize by energy scale =(distance scale)$^{-1}$



Classify terms in Hamiltonian as "relevant" or "irrelevant"

RG Flow between CFTs



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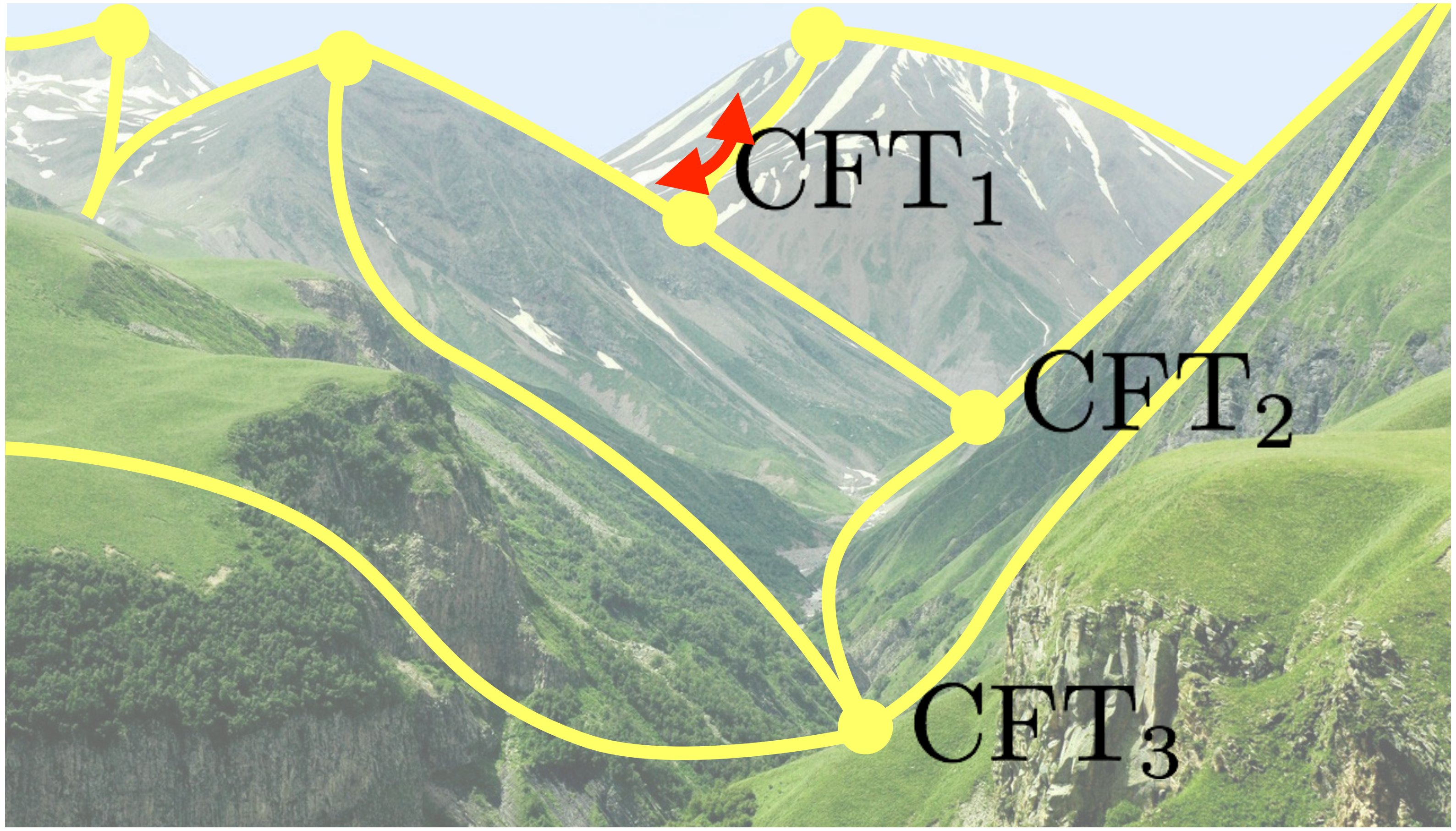

Add **irrelevant** term to CFT: RG flow pushes you back

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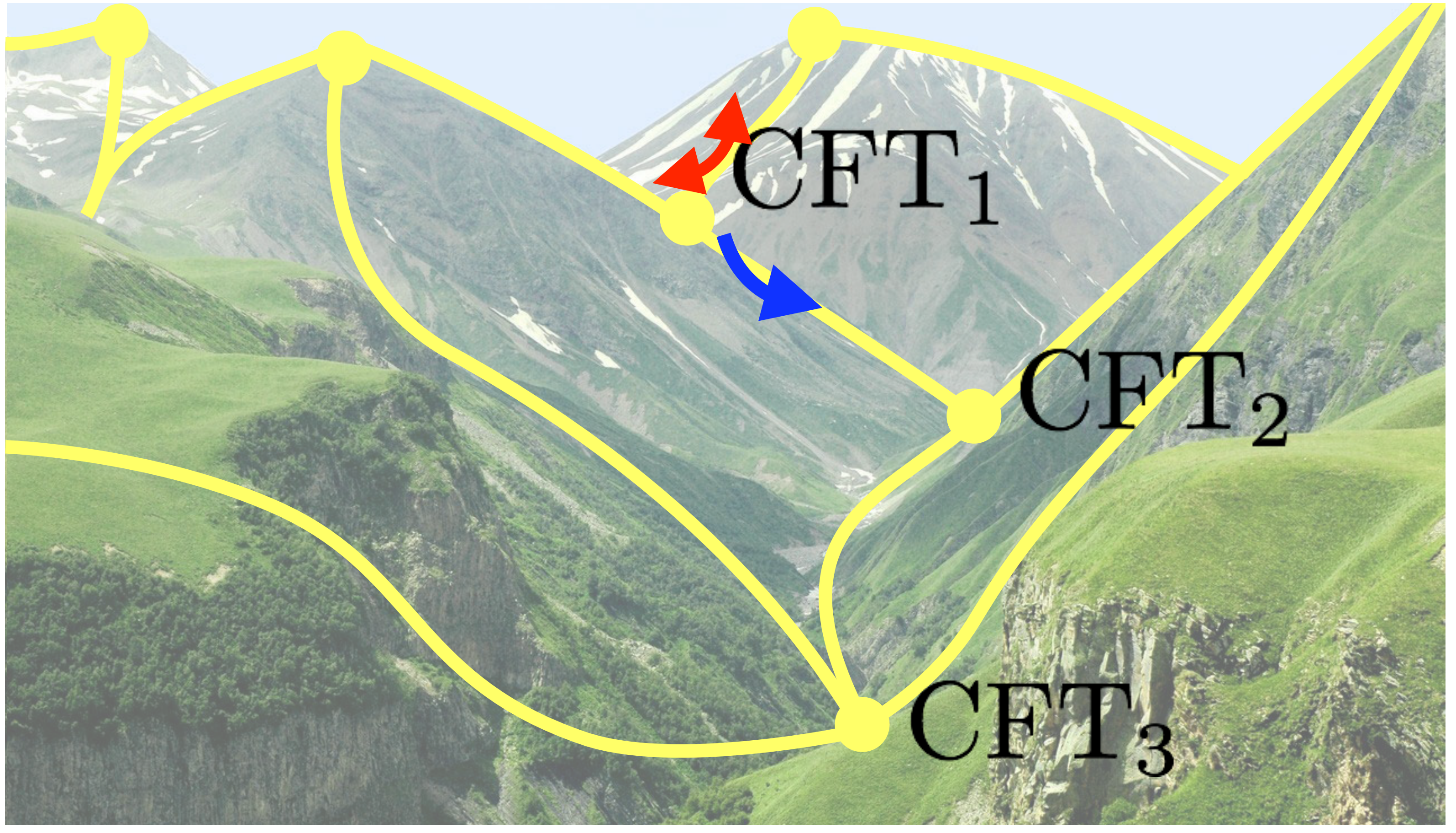


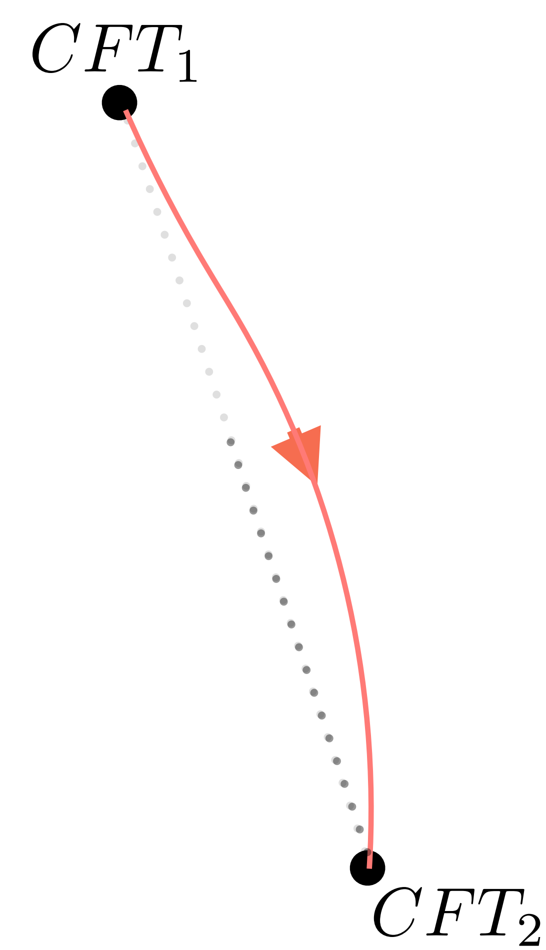
Add **irrelevant** term to CFT: RG flow pushes you back

Add ***relevant*** term to CFT: RG flow pushes you to new CFT

Grand Bootstrap Goal: Solve QFT in 2 easy steps

Step 1: Solve CFT

Step 2: Solve RG flow



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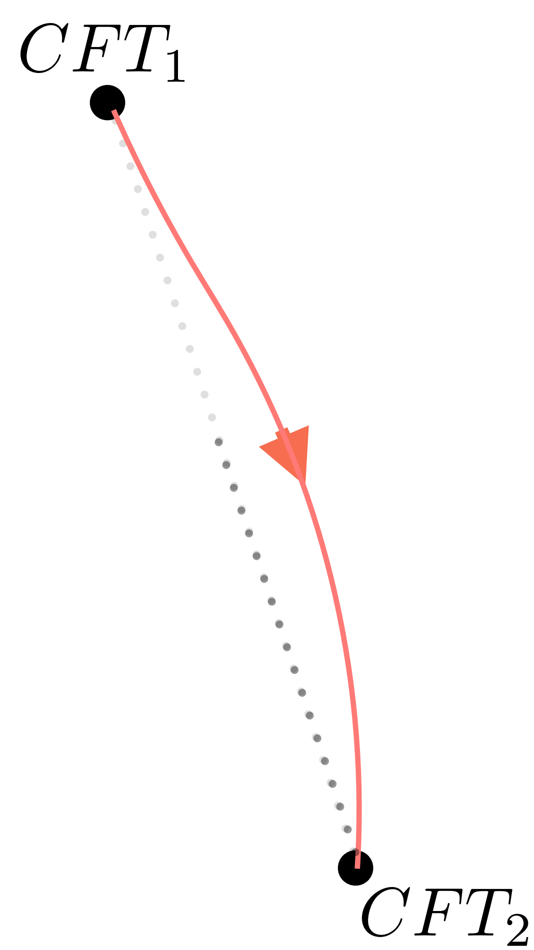

Conformal Bootstrap

1970's: Polyakov shows that form of 2- and 3-point functions in CFTs are fixed by symmetry

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\rangle = \frac{c_{ijk}}{x_{12}^{\Delta_{ij,k}}\, x_{13}^{\Delta_{ik,j}}\, x_{23}^{\Delta_{jk,i}}}$$

and proposes bootstrap equation as consistency conditions on CFTs

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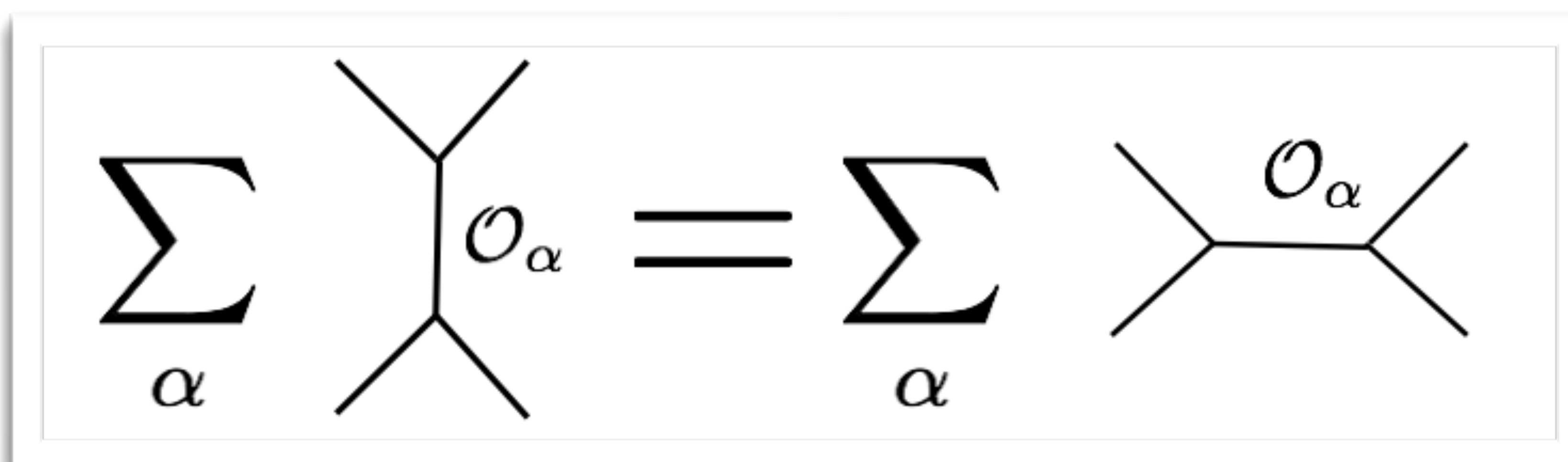
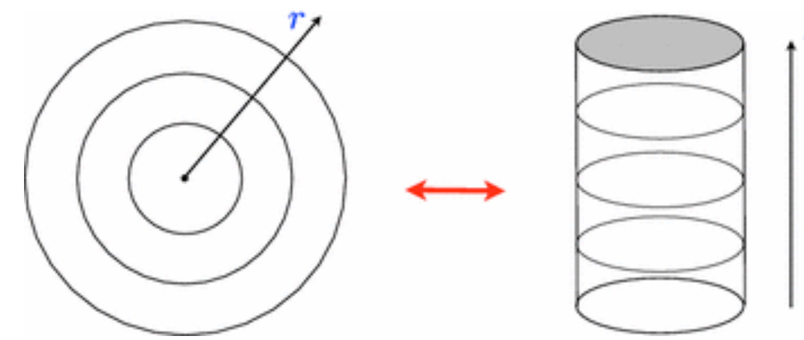


Conformal Bootstrap

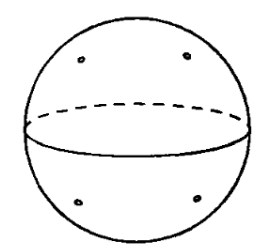
Key concepts behind bootstrap:

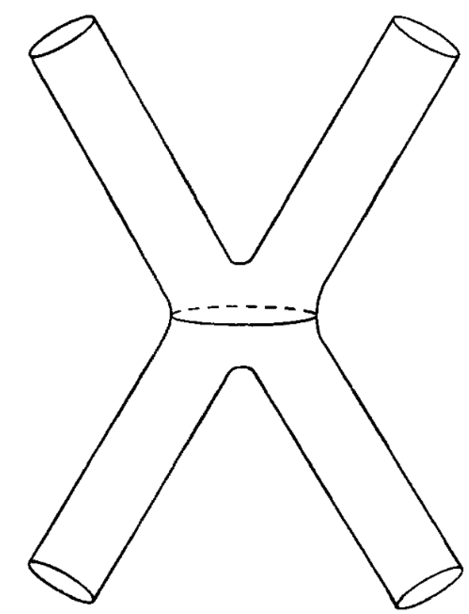
Radial quantization: conformal map from the plane to the (sphere) x (time)



Implies state-operator correspondence:

"states = local operators"





Infinite volume dynamics are controlled by finite volume system with a *discrete* set of "CFT" data

$$\{\Delta_i, c_{ijk}\}$$

Conformal Bootstrap

Key concepts behind bootstrap:

Operator Product Expansion: $\mathcal{O}_i \times \mathcal{O}_j = \sum_k c_{ijk}\mathcal{O}_k$

Start with insertion of two operators

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E.g. Spherical harmonics

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E.g. ~~Spherical harmonics~~ Conformal Blocks

Conformal Bootstrap

Key concepts behind bootstrap: **Bootstrap Equation**

Consider inserting 4 operators.

Powerful constraint: we can look at the wavefunction of the system on any surface and decompose into "conformal blocks" = "conformal spherical harmonics"

$$\begin{array}{cc} \rho(x_1) & \rho(x_4) \\ \times & \times \\ & \\ \times & \times \\ \rho(x_2) & \rho(x_3) \end{array}$$

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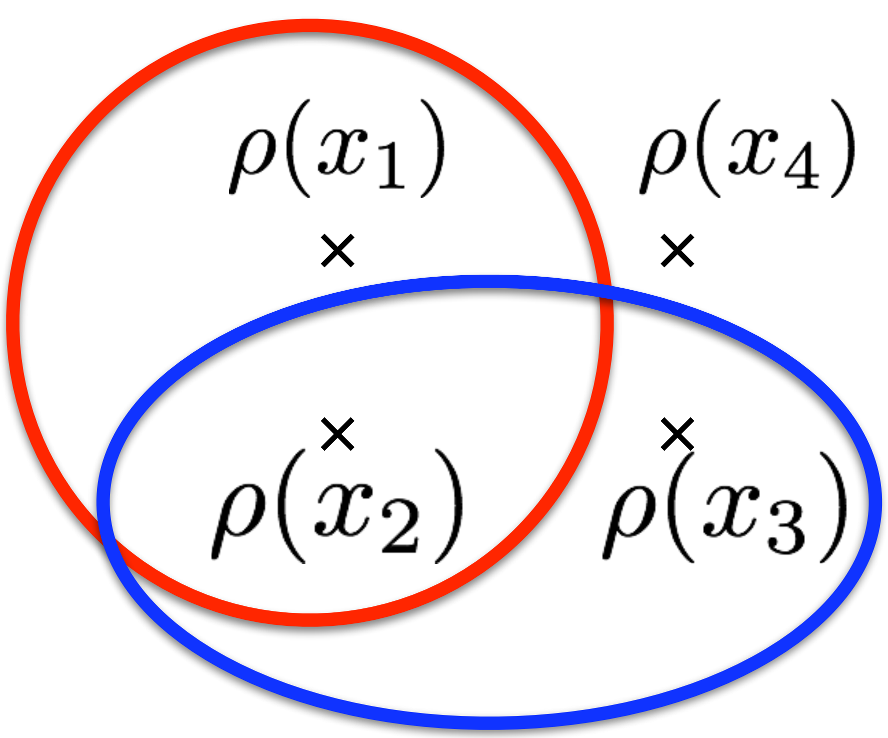

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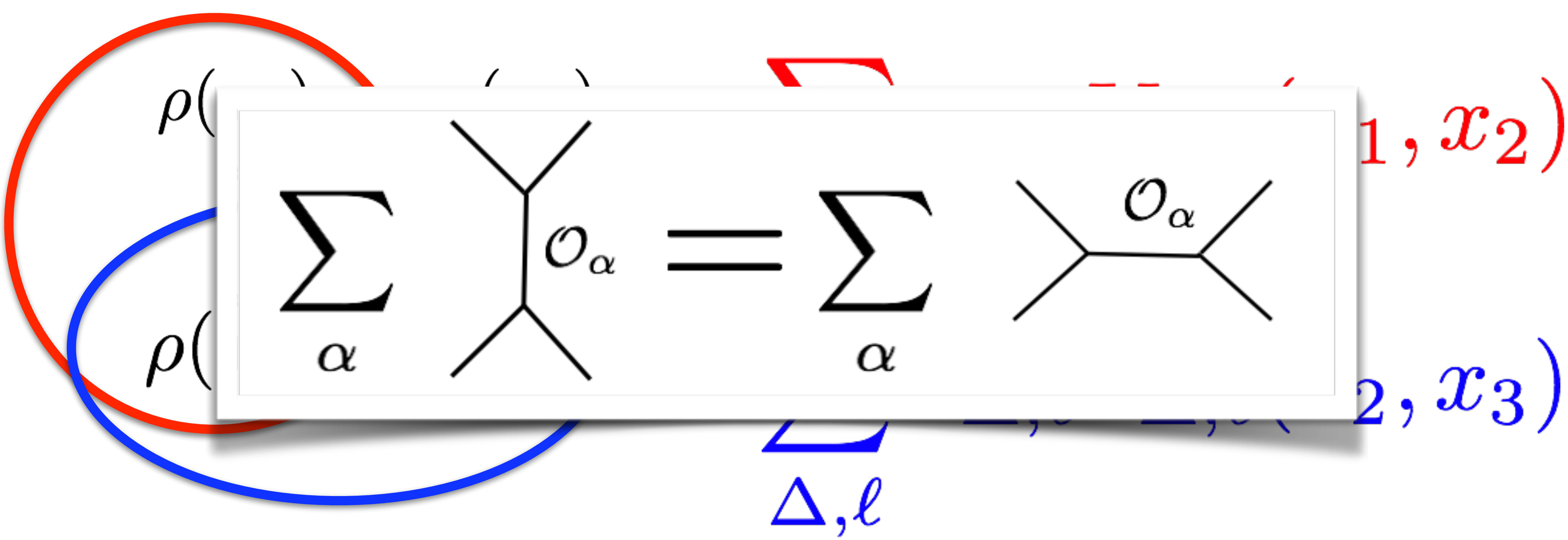
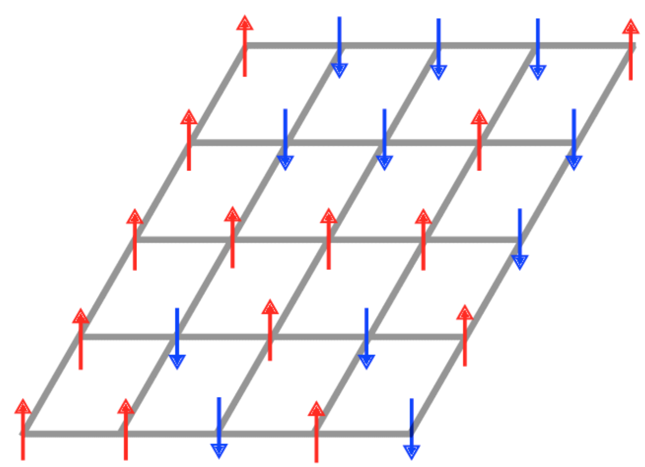


Conformal Bootstrap

1980's: Belavin, Polyakov, & Zamolodchikov show how to use these ideas to solve special* 2d CFTs

eg 2d Ising model



*Crucially, these special CFTs have only a *finite* amount of "CFT data" $\{\Delta_i, c_{ijk}\}$

But most 2d CFTs, and all CFTs at d>2, have an infinite amount of CFT data

Conformal Bootstrap Rebirth

2008: Rattazzi, Rychkov, Tonni, & Vichi

(Also, 2009: Simeon Kellerman - Modular Bootstrap)

Key idea: prove bounds by contradiction

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MAYBE!

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Conformal Oracle from Bootstrap Equation

First, do some trivial reorganization of bootstrap equation

$$\sum_{\Delta,\ell} a_{\Delta,\ell} Y_{\Delta,\ell}(x_1, x_2) = \sum_{\Delta,\ell} a_{\Delta,\ell} Y_{\Delta,\ell}(x_2, x_3)$$

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Rewrite as

$$0 = \sum_{\Delta,\ell} a_{\Delta,\ell} \overbrace{F(x_i)}^{Y_{\Delta,\ell}(x_1,x_2) - Y_{\Delta,\ell}(x_2,x_3)}$$

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Conformal Bootstrap Rebirth

Next, assume "partial CFT data" $\{\Delta_i, c_{ijk}\}$ is correct, try to rule it out by proving a contradiction

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Idea: numerically find a linear functional $\rho : F_{\Delta,\ell} \to \mathbb{R}$ such that

$\rho(F_{\text{vac}}) = 1$ and $\rho(F_{\Delta,\ell}) \geq 0$ for all Δ, ℓ in $\{\Delta_i, c_{ijk}\}$

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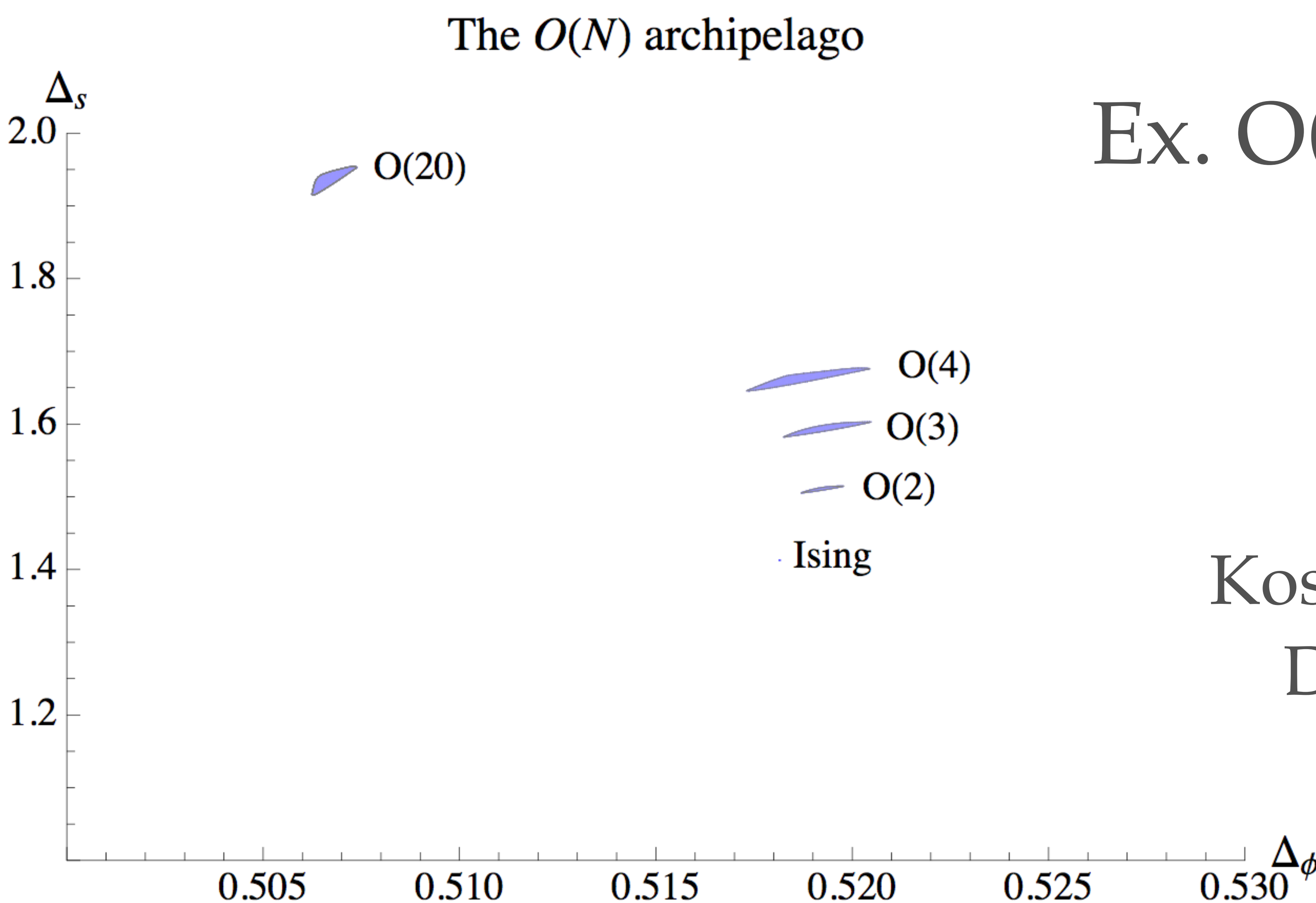
If you can find such a functional, then, acting on the bootstrap equation, it implies

$0 \geq 1$ Contradiction!

3d Islands



Ex. O(N) models in d=2+1

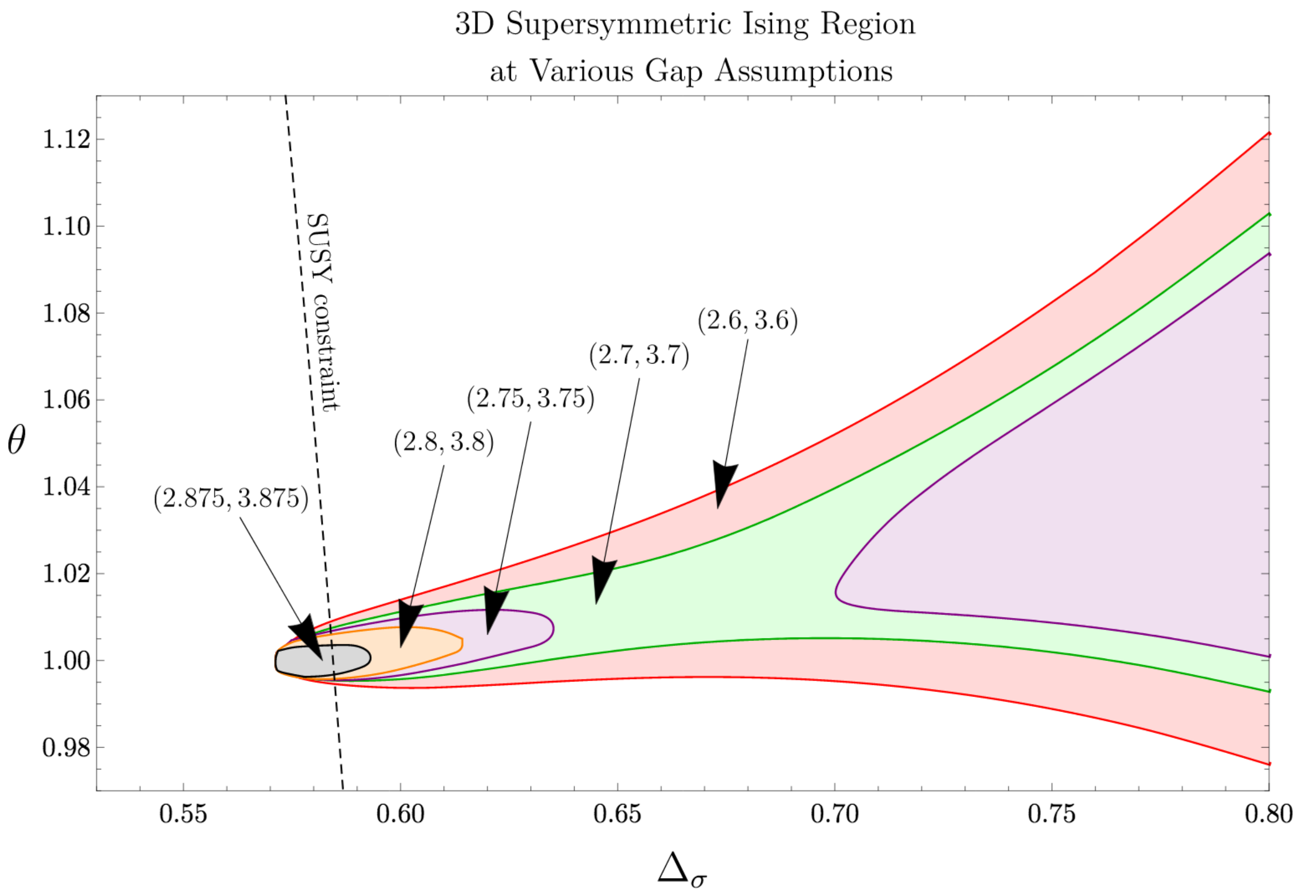
Kos, Poland, Simmons-Duffin, Vichi, 2015

Ising "island" almost too small to see by eye

At this point, can extract many OPE coefficients as well

3d Islands

However, it's still a bit of an art to "carve out" islands



Atanasov, Hillman, &Poland, 2018

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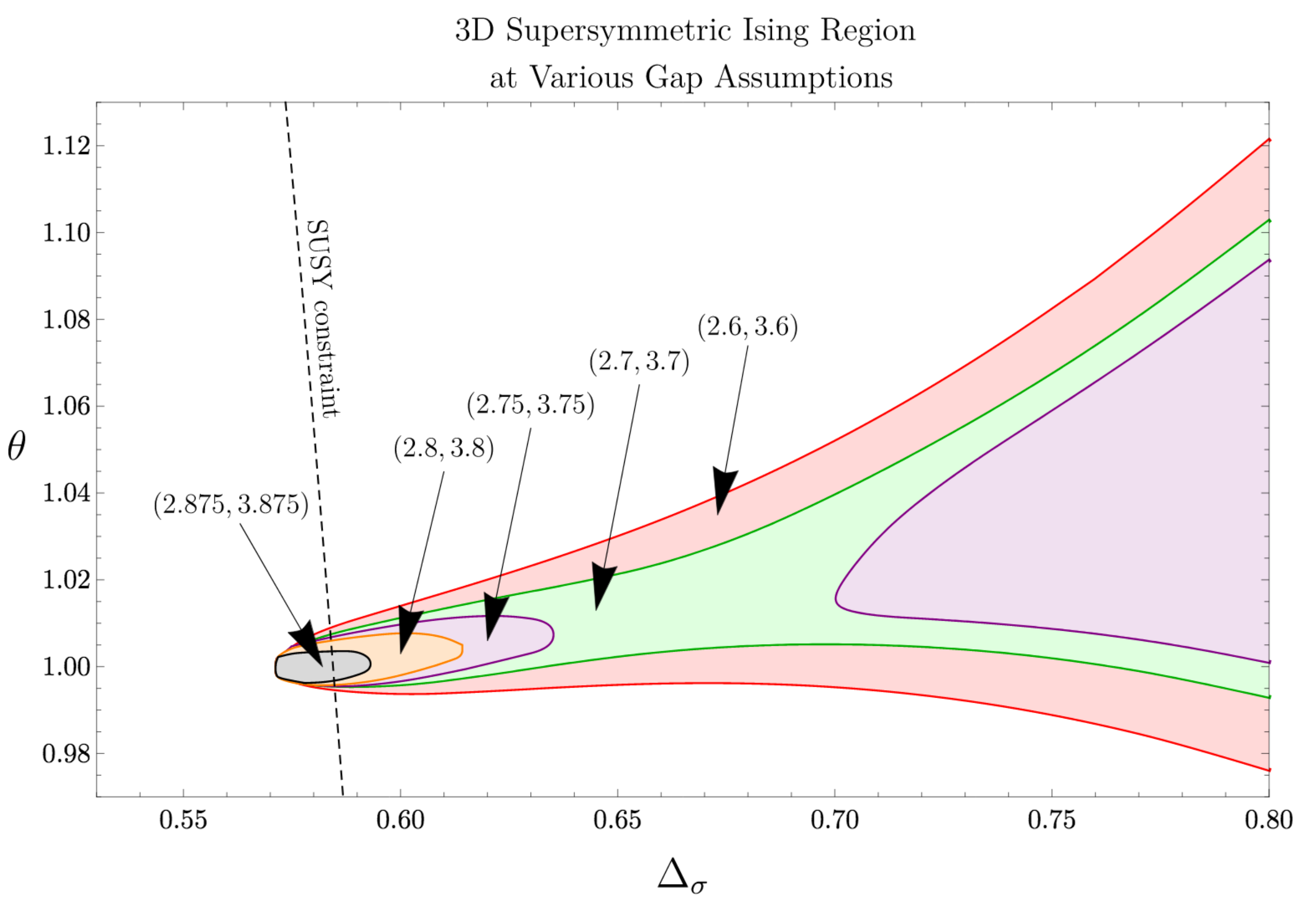
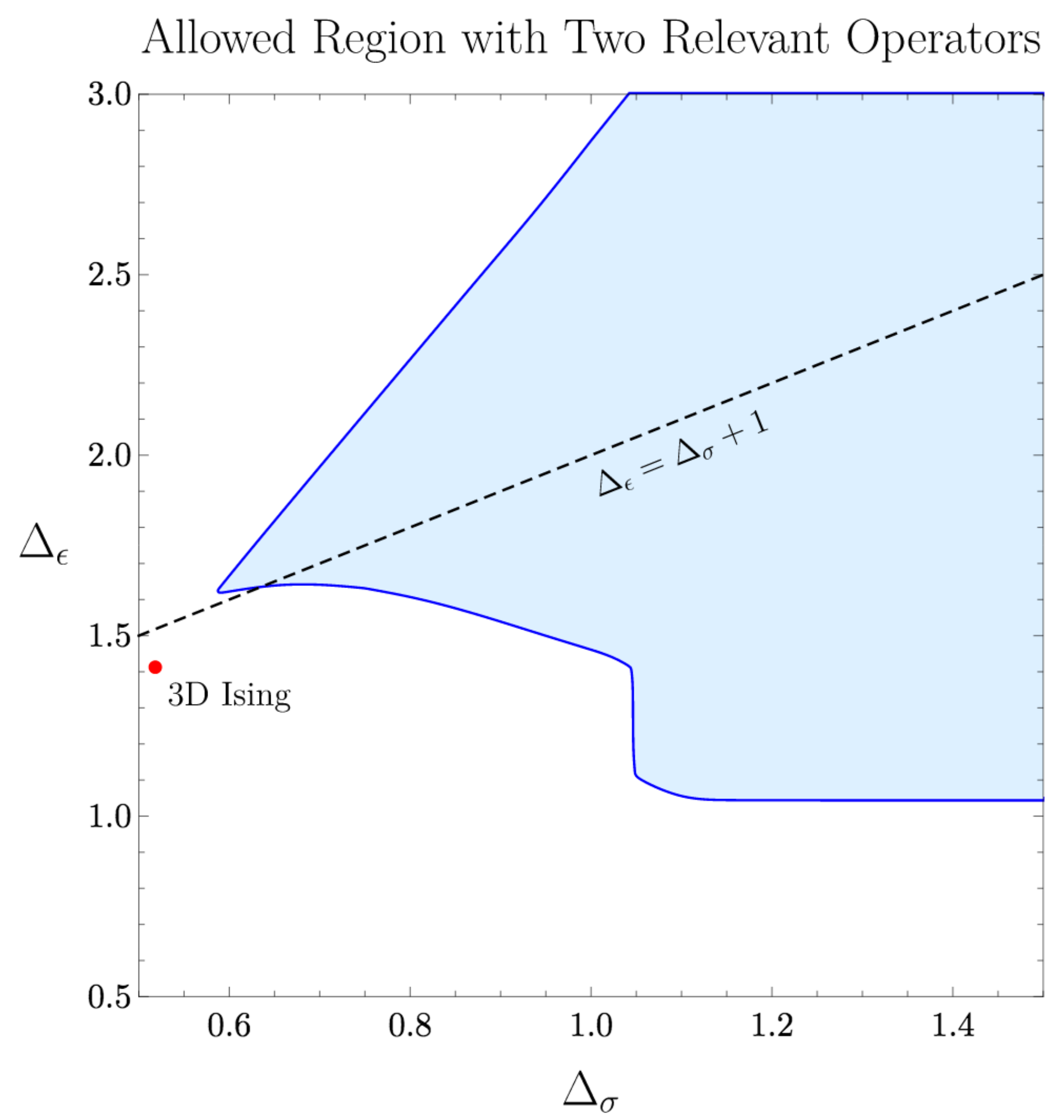


and not fully understood why it works well sometimes but not others



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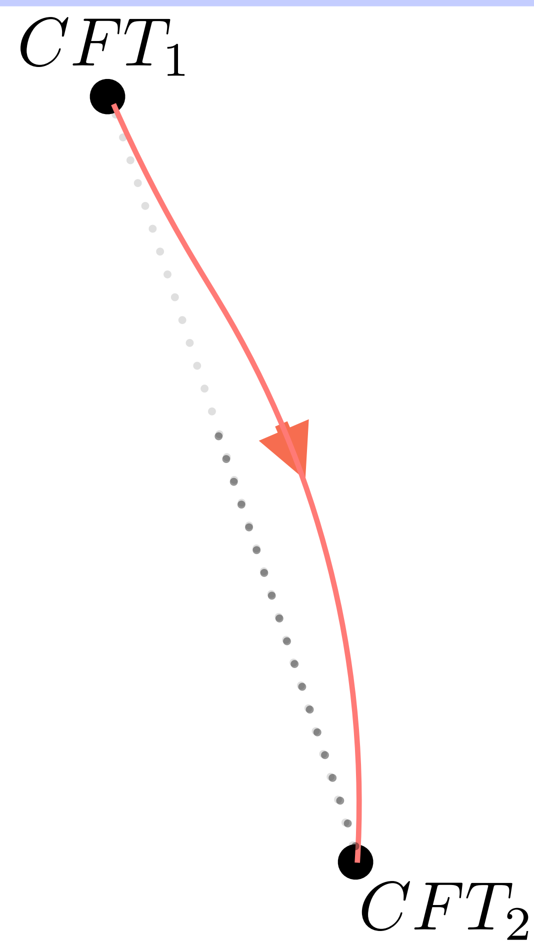

What about RG flows?

RG flow triggered by relevant deformation

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solved in "step 1" *"interaction" term*

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RG flow triggered by relevant deformation

$$H = H_{\mathrm{CFT}} + \lambda \mathcal{O}_{\mathrm{Relevant}}$$

solved in "step 1" *"interaction" term*

Idea: Do ***exact*** diagonalization on *H* in a basis of eigenstates of H_{CFT} (Rayleigh-Ritz)

$$\langle \mathcal{O}_1 | H | \mathcal{O}_2 \rangle$$

Matrix elements just depend on UV CFT data

Truncation Warm-up: Anharmonic Oscillator in QM

$$\mathcal{L} = \frac{1}{2}\dot{x}^2 - \frac{1}{2}m^2x^2 - \overbrace{gx^4}^{V} \longrightarrow H = H_{\text{SHO}} + V$$

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$$V \sim g(a + a^\dagger)^4$$

Basis: $|n\rangle = \frac{1}{\sqrt{n!}}(a^\dagger)^n|0\rangle \qquad \langle n|H|m\rangle = \delta_{nm}E_n^{(\rm SHO)} + \langle n|V|m\rangle$

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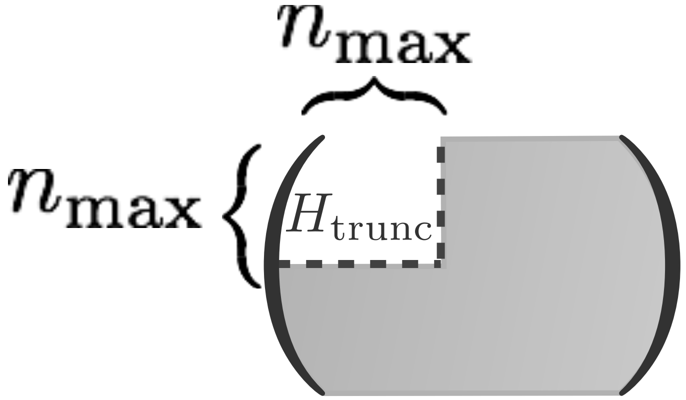
Take finite $n_{\max}$ and diagonalize

$n_{\max}$ x $n_{\max}$ matrix

Eigenvalues converge quickly

as a function of $n_{\max}$



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Take finite n_{max} and diagonalize n_{max} x n_{max} matrix

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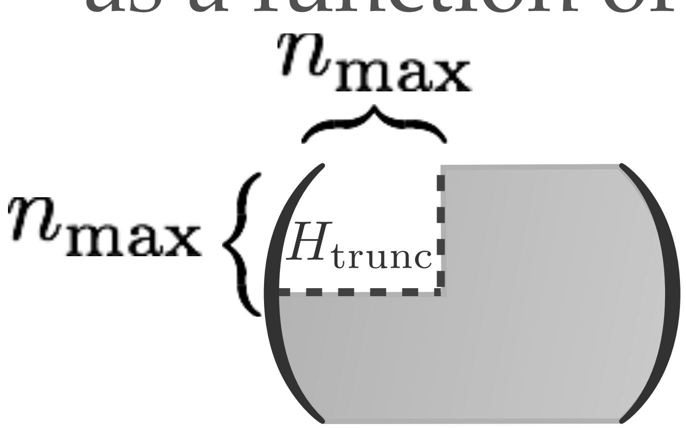
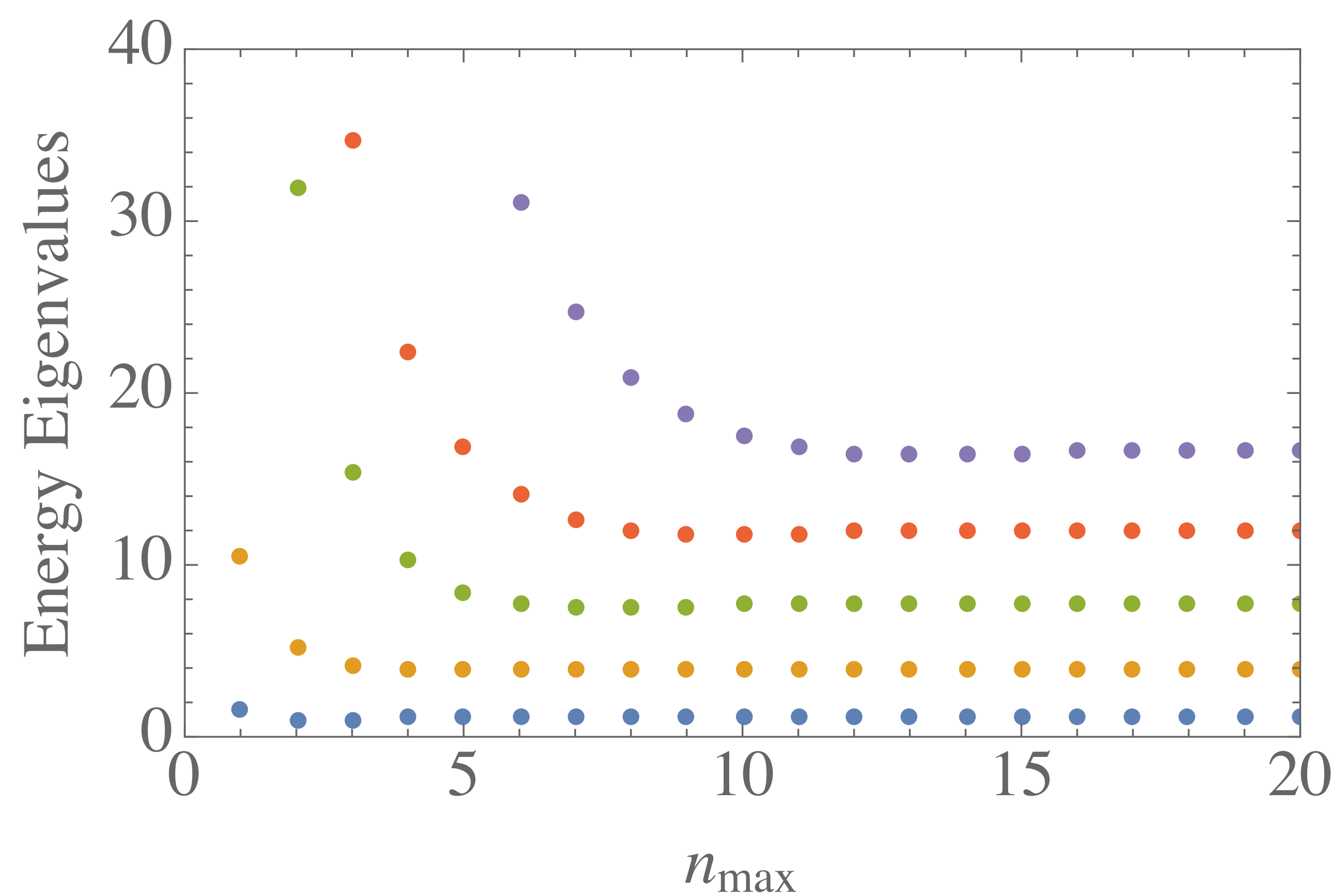
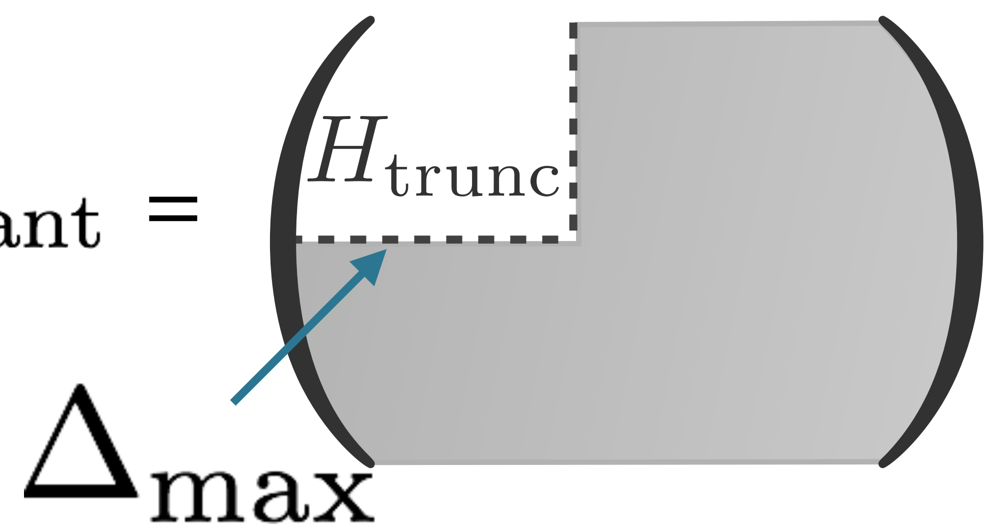




Conformal Truncation

Same idea, applied to QFT Hamiltonian

$$H = H_{\mathrm{CFT}} + \lambda \mathcal{O}_{\mathrm{Relevant}} =$$

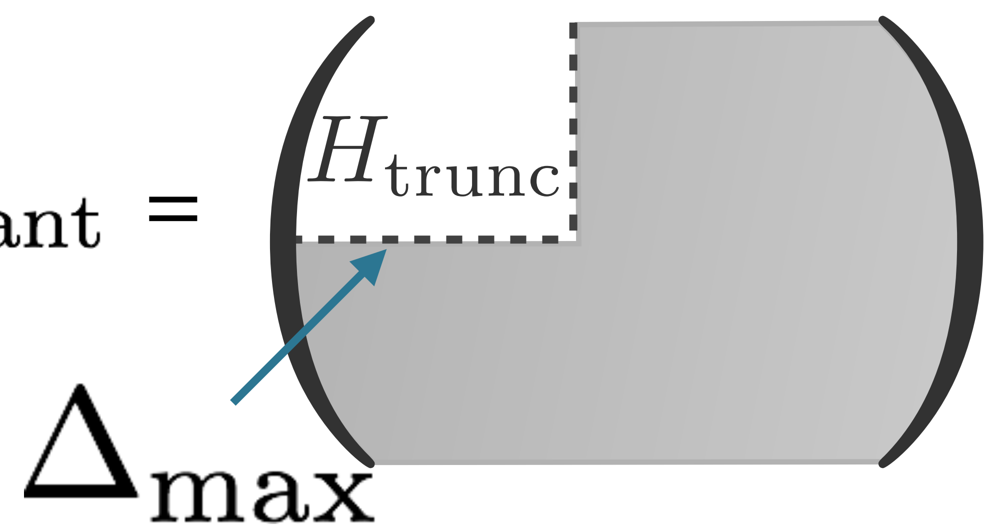


Idea: keep only states with dimension below Δ_{max}

Conformal Truncation

Same idea, applied to QFT Hamiltonian

$$H = H_{\text{CFT}} + \lambda \mathcal{O}_{\text{Relevant}} =$$



Idea: keep only states with dimension below $\Delta_{\max}$

$\langle \mathcal{O}_1 | H | \mathcal{O}_2 \rangle$ Matrix elements just depend on UV CFT data

$\langle \mathcal{O}_1 | H_{\text{CFT}} | \mathcal{O}_2 \rangle$ determined by CFT dimensions Δ_i

$\langle \mathcal{O}_1 | \mathcal{O}_{\text{Relevant}} | \mathcal{O}_2 \rangle$ determined by CFT OPE coefficients c_{ijk}

Conformal Truncation

1990 : Zamolodchikov & Yurov apply conformal truncation to deformations of solvable 2d CFTs in finite volume

More recent work:

ϕ^4 theory in 2d

finite volume:

Hogervorst, Rychkov, Van Rees, Vitale '14

2d + 3d

infinite volume:

Anand, Genest, Katz, Khandker, Walters '17

2d QCD @ finite N_c

Katz, Tavares, Xu '14

3d U(1) Chern-Simons gauge theory at large N_f

ALF, Katz, Vitale, Walters '18

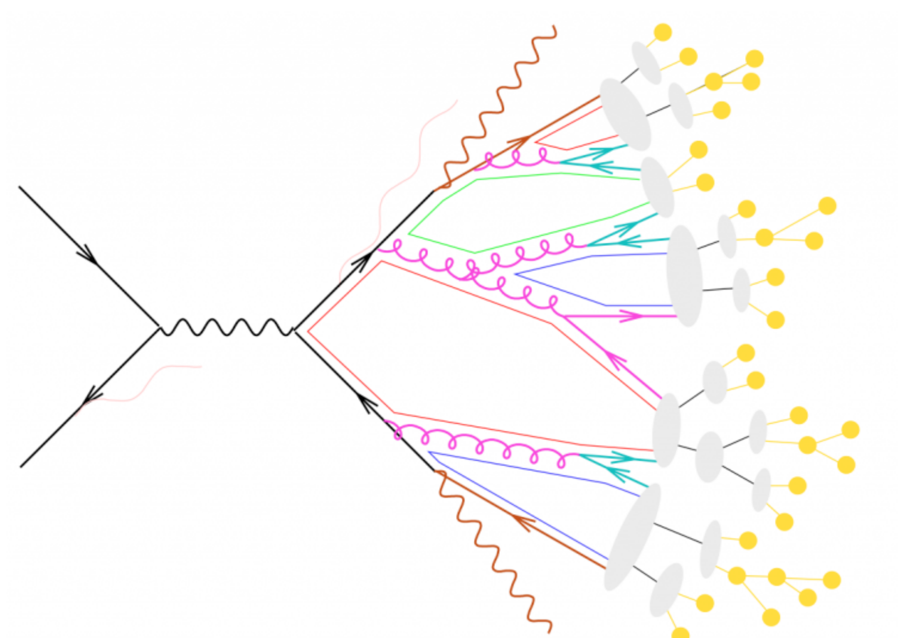
Conformal Truncation

Because you diagonalize the Hamiltonian, you get eigenvalues ***and*** eigenvectors

This allows one to compute spectral functions, which encode correlation functions

$$\langle J^\mu(p) J^\nu(-p)\rangle = (q^2\eta^{\mu\nu} - q^\mu q^\nu)\int_0^\infty ds\,\frac{\rho(s)}{p^2 - s + i\epsilon}$$

and scattering amplitudes

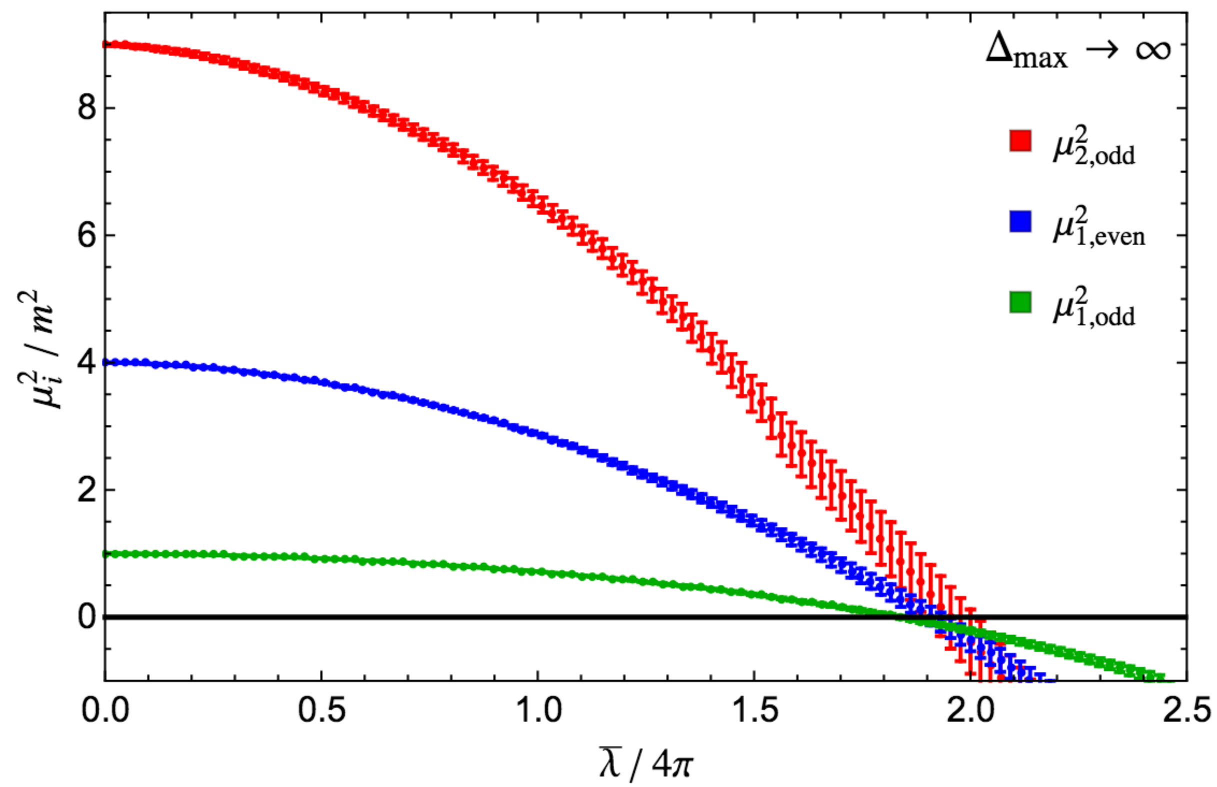


$$\sigma(e^+e^- \to \text{hadrons}) = -\frac{(4\pi\alpha)^2}{s}\rho(s)$$

Conformal Truncation

Example $d = 2$ $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$ $\bar{\lambda} = \frac{\lambda}{m^2}$

Eigenvalues:
mass spectrum as a
function of coupling



Anand et al. '17

Conformal Truncation

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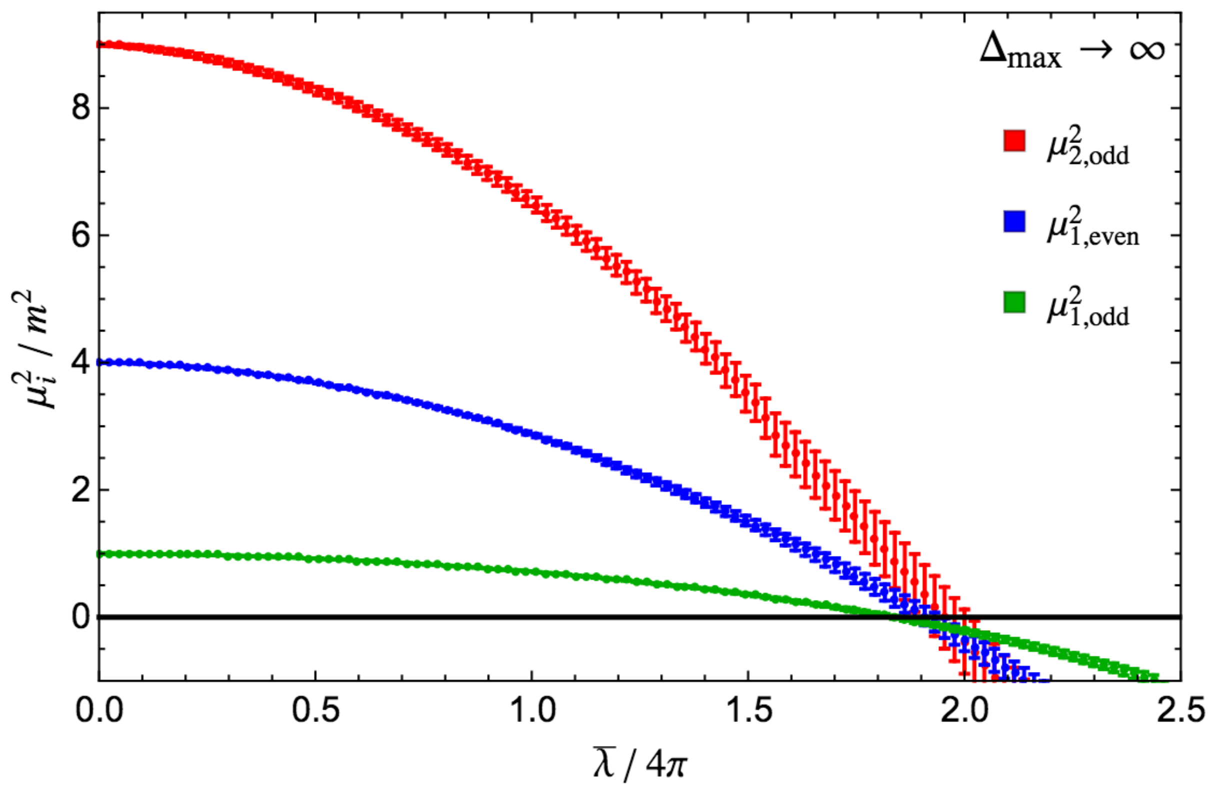
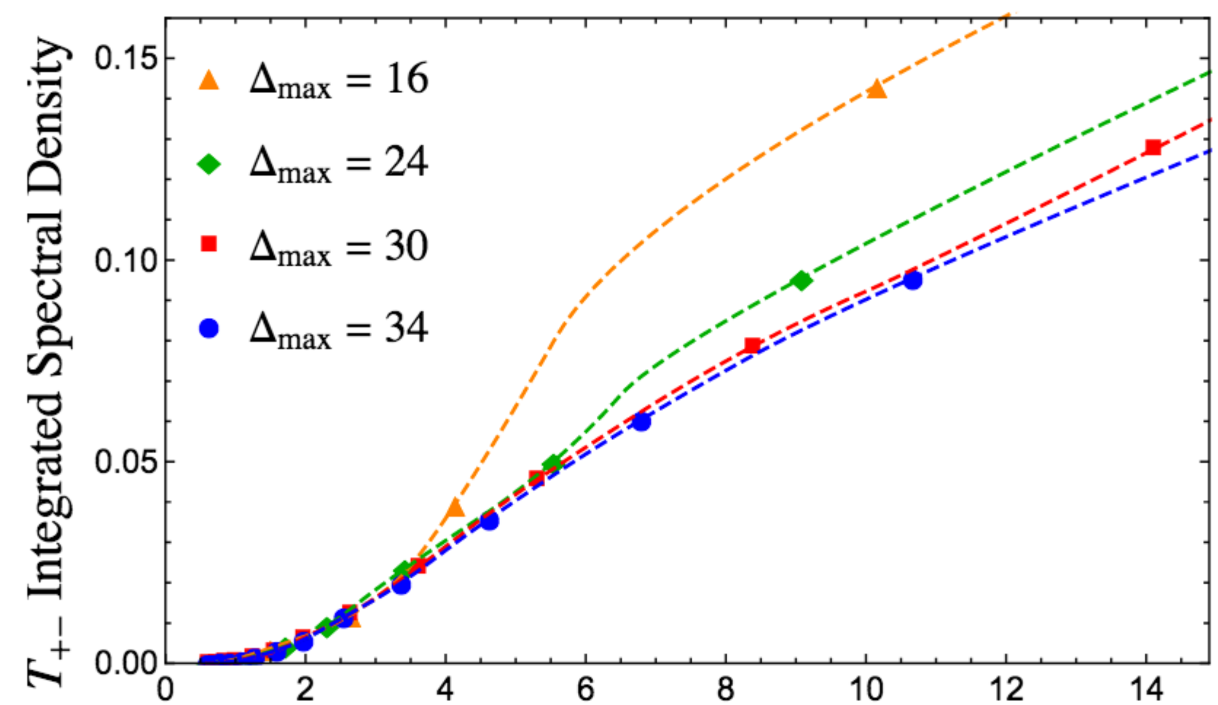


Eigenvectors:
spectral function of stress tensors as a function of momentum scale



Anand et al. '17

$$\mathcal{I}_{T_{+-}}(\mu) \equiv \int_0^{\mu} d\mu'^2 \rho_{T_{+-}}(\mu')$$

But many challenges remain…

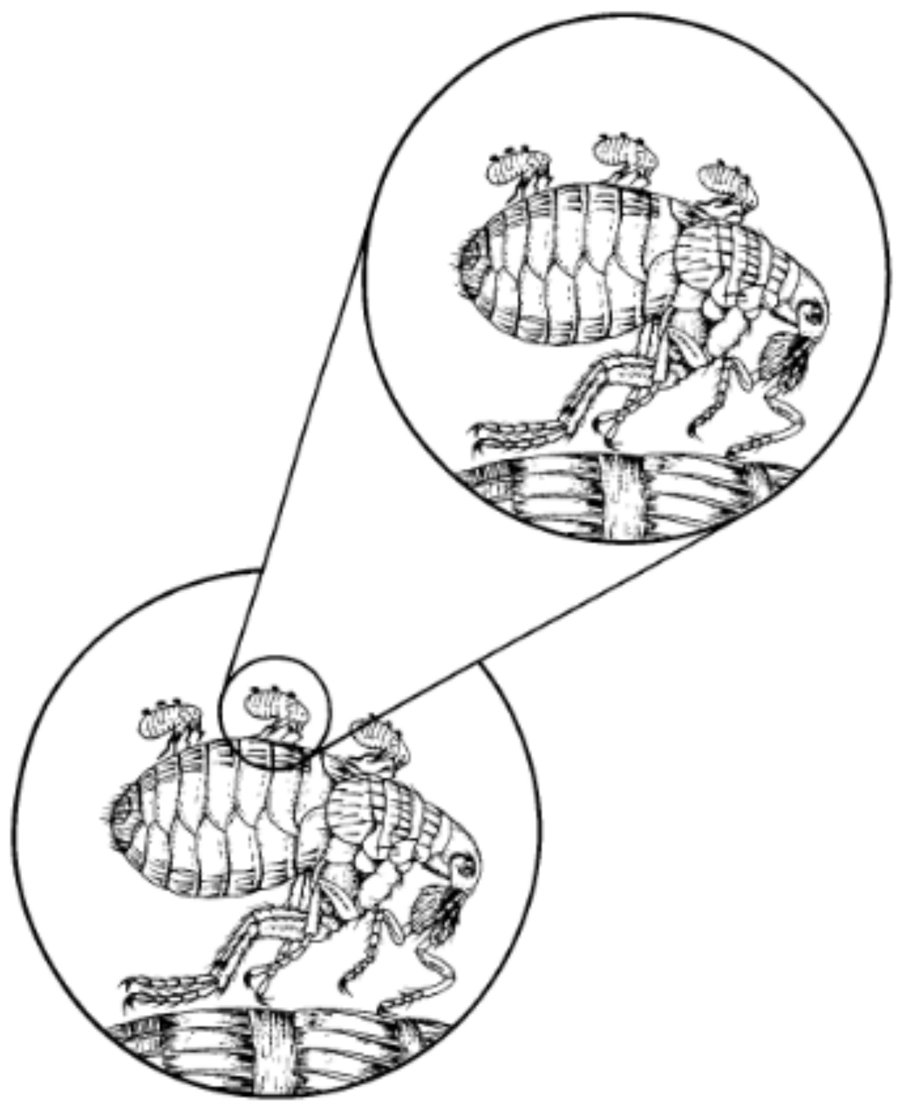
1) At present, bootstrap cannot determine CFT data for most CFTs - only some “lucky” CFTs that happen to be at the edge of numeric bounds

2) Conformal truncation requires one to solve the UV CFT - can only use special solvable CFTs (e.g. free theories & integrable models)

3) Requires intensive numerical work - can more efficient methods be invented?

4) Etc.

THE END

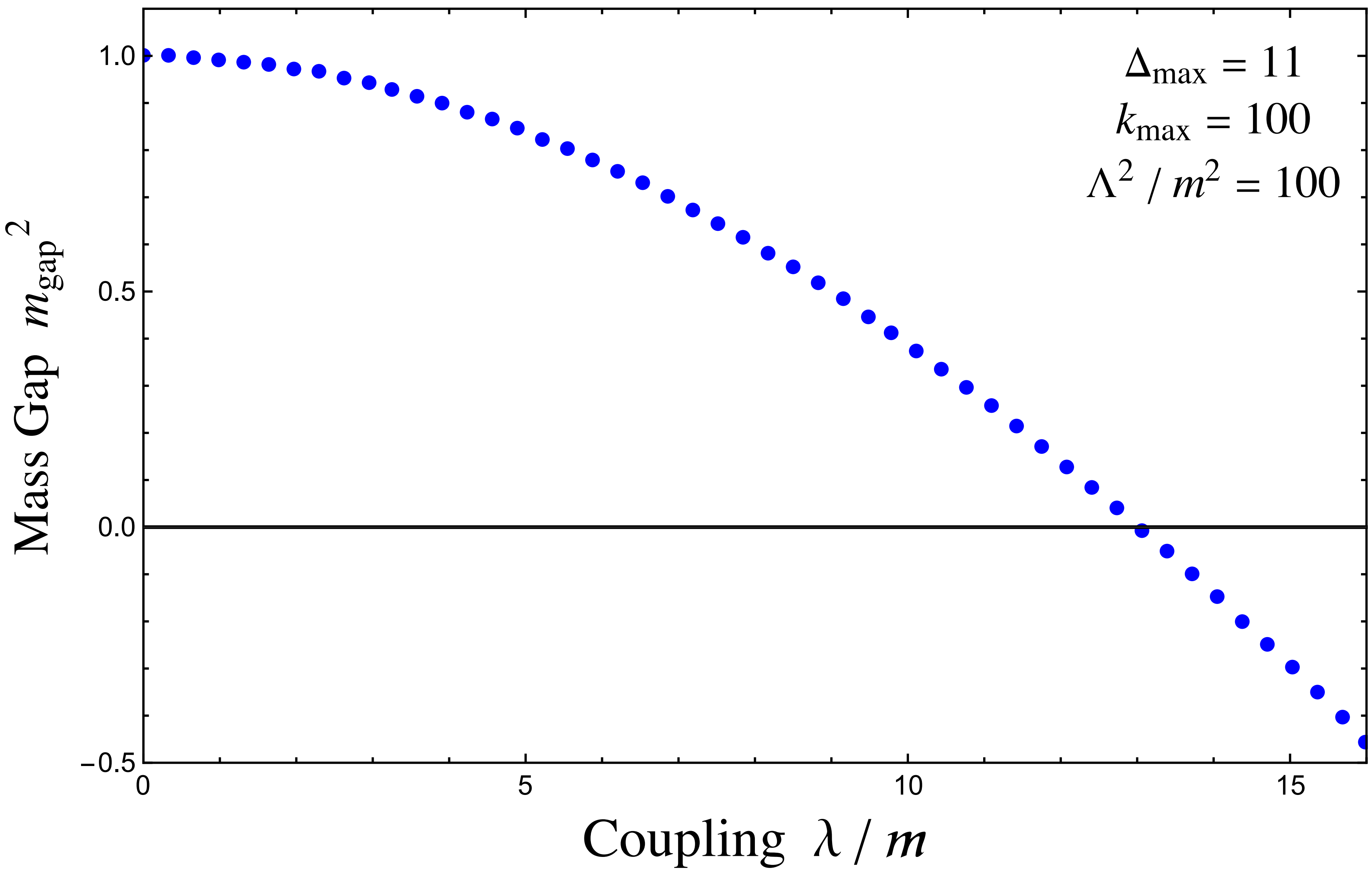


Backup Slides

$$d = 3 \quad \mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

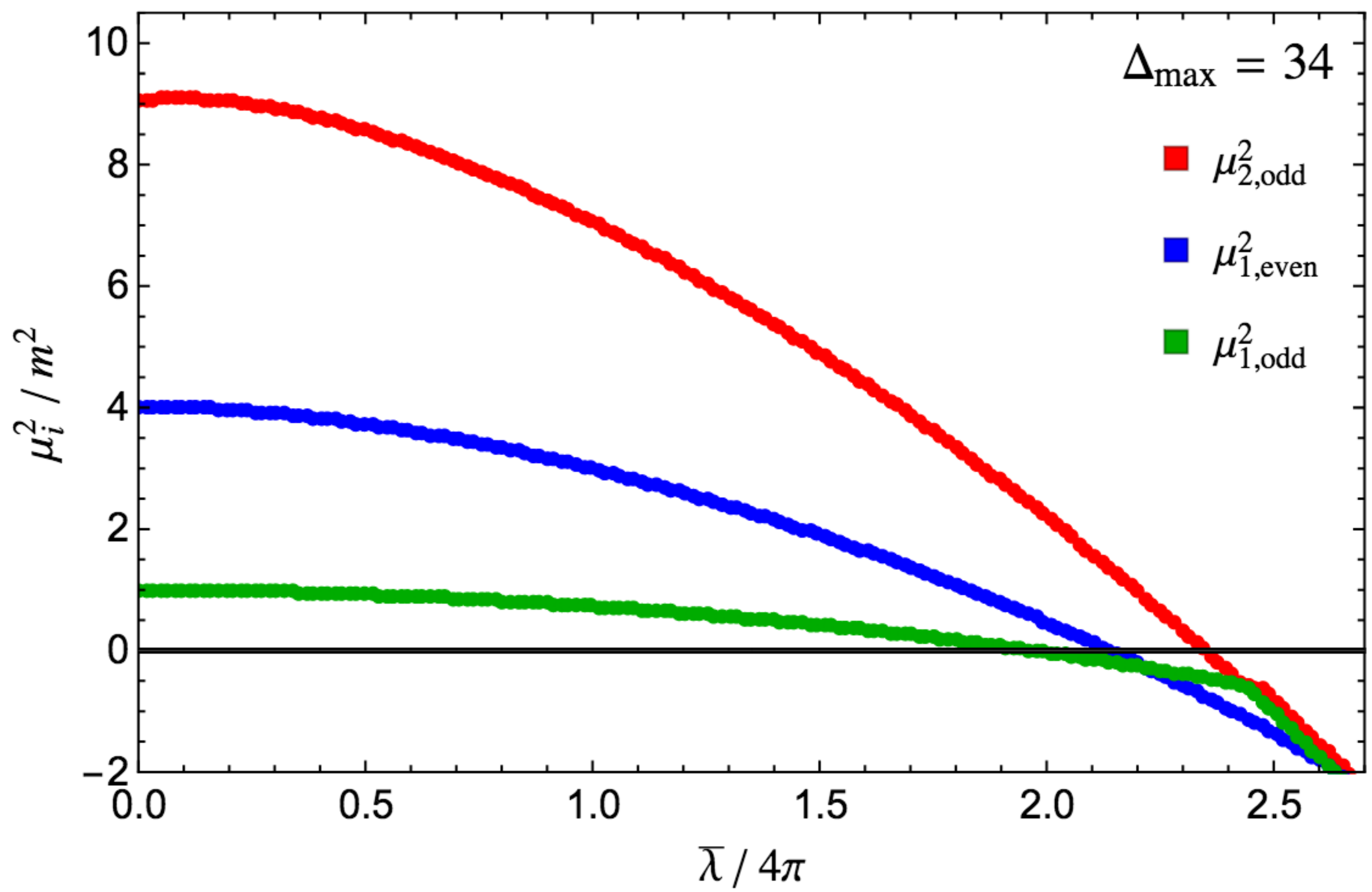
3d Ising

Preliminary



M. Walters

$$d = 2 \quad \mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \qquad \bar{\lambda} = \frac{\lambda}{m^2}$$



Anand et al. '17

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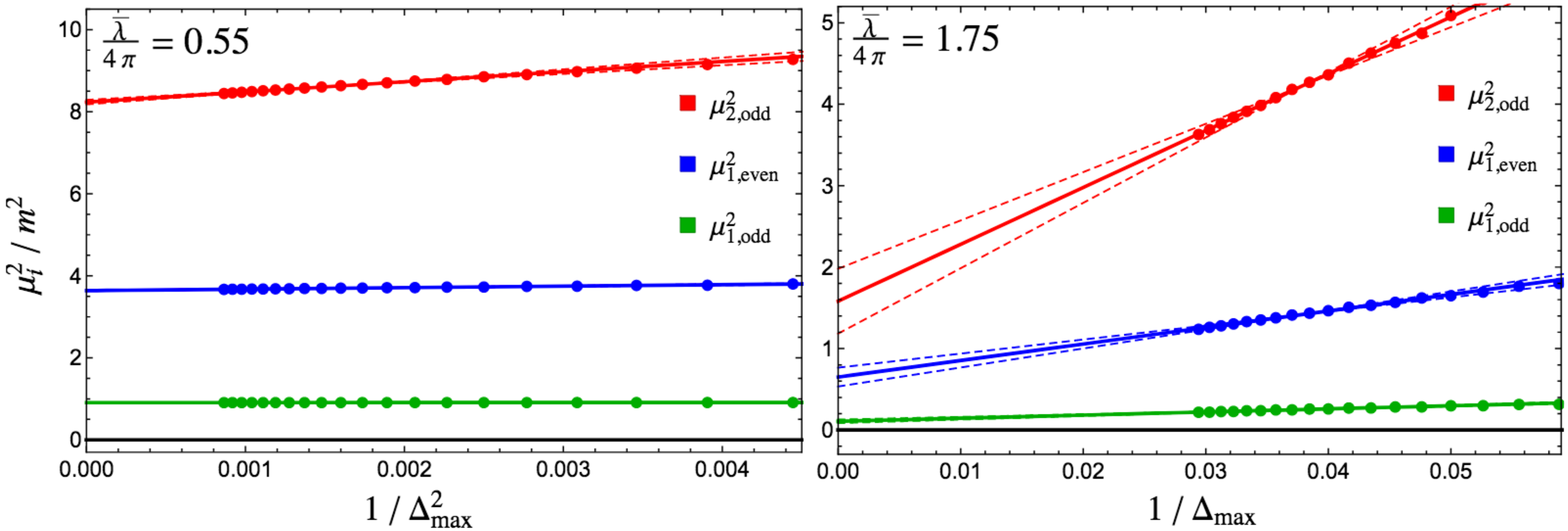

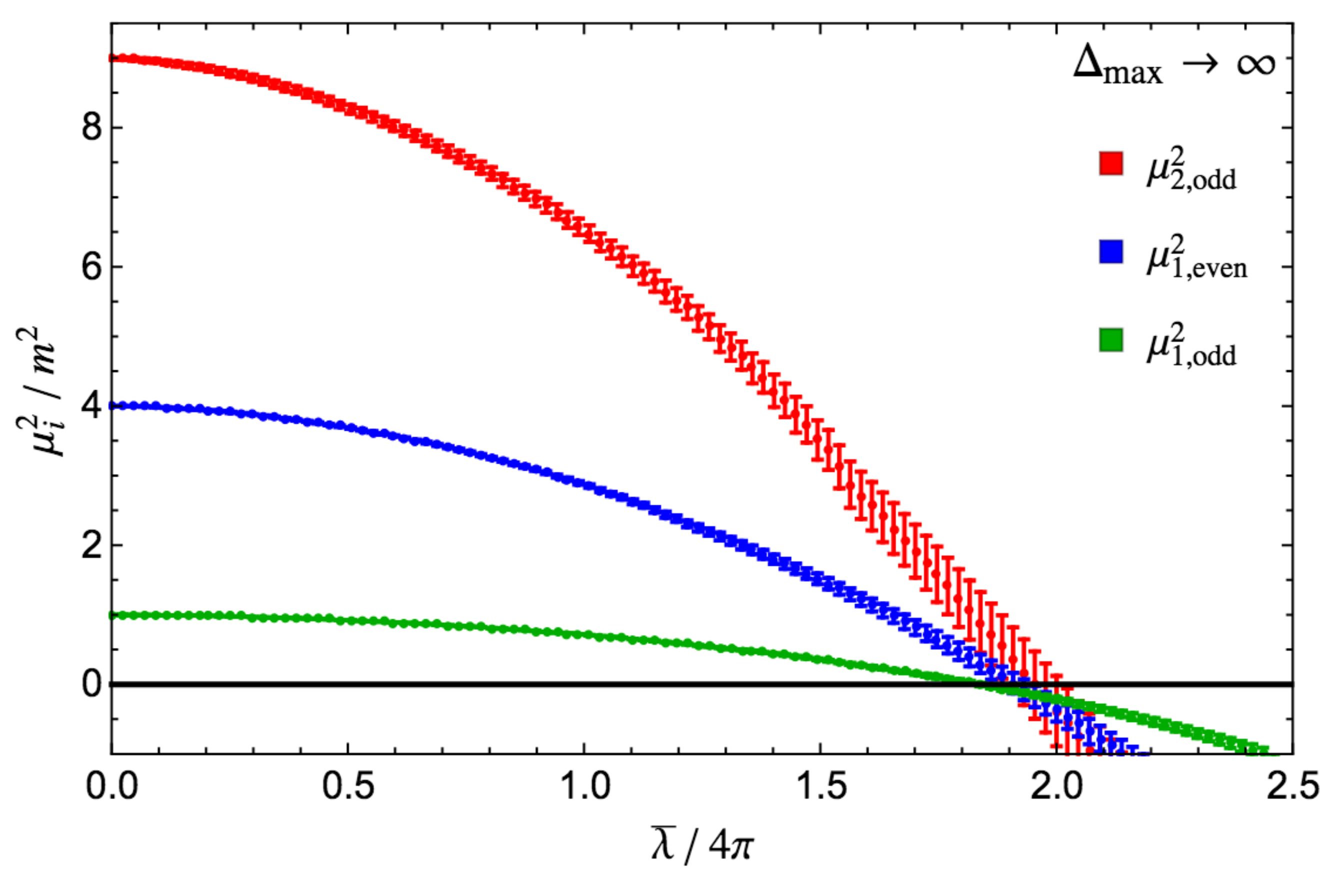

Anand et al. '17

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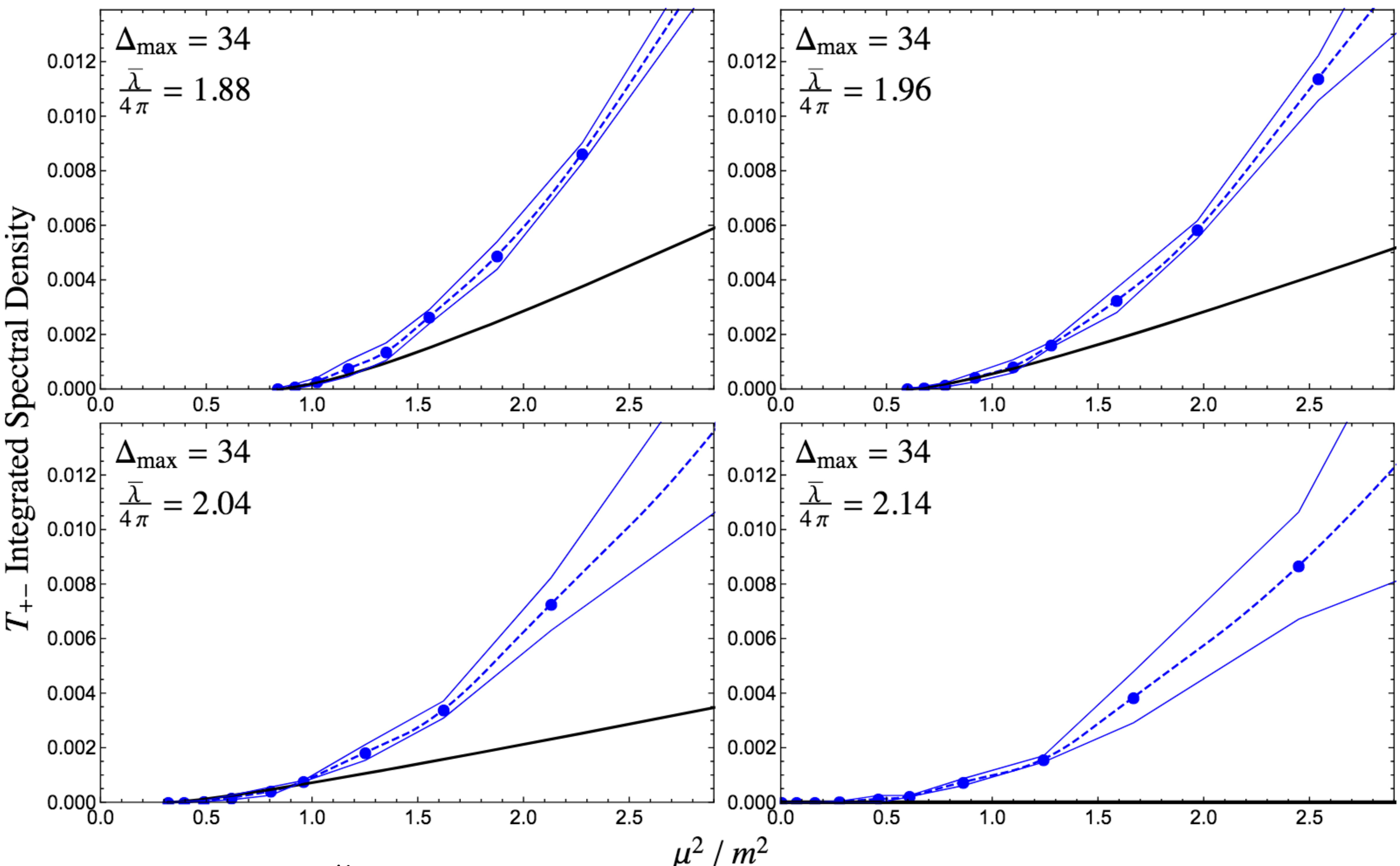
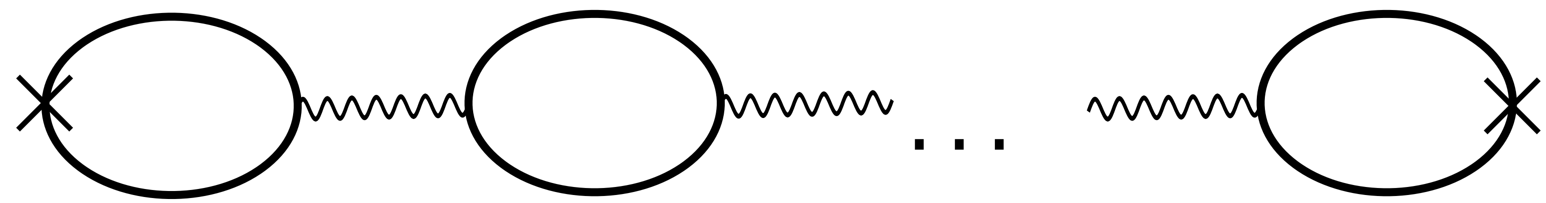


$$\mathcal{I}_{T_{+-}}(\mu) \equiv \int_0^\mu d\mu'^2 \rho_{T_{+-}}(\mu')$$

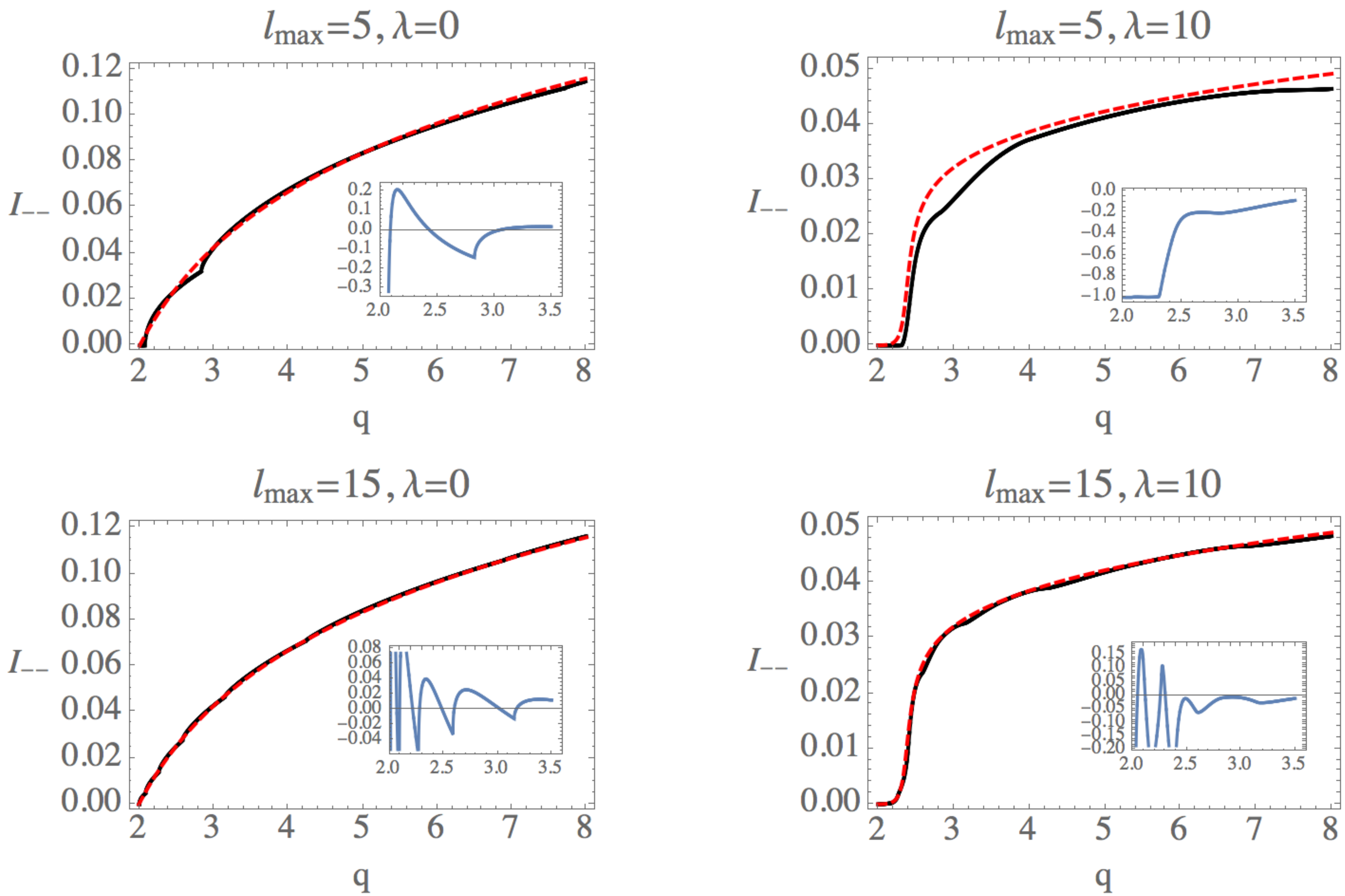
3d Chern-Simons $N_f = \infty$ $\mathcal{L} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi + \frac{k}{4\pi} a da - m_a a^2$

$$\langle J_\mu J_\nu \rangle =$$



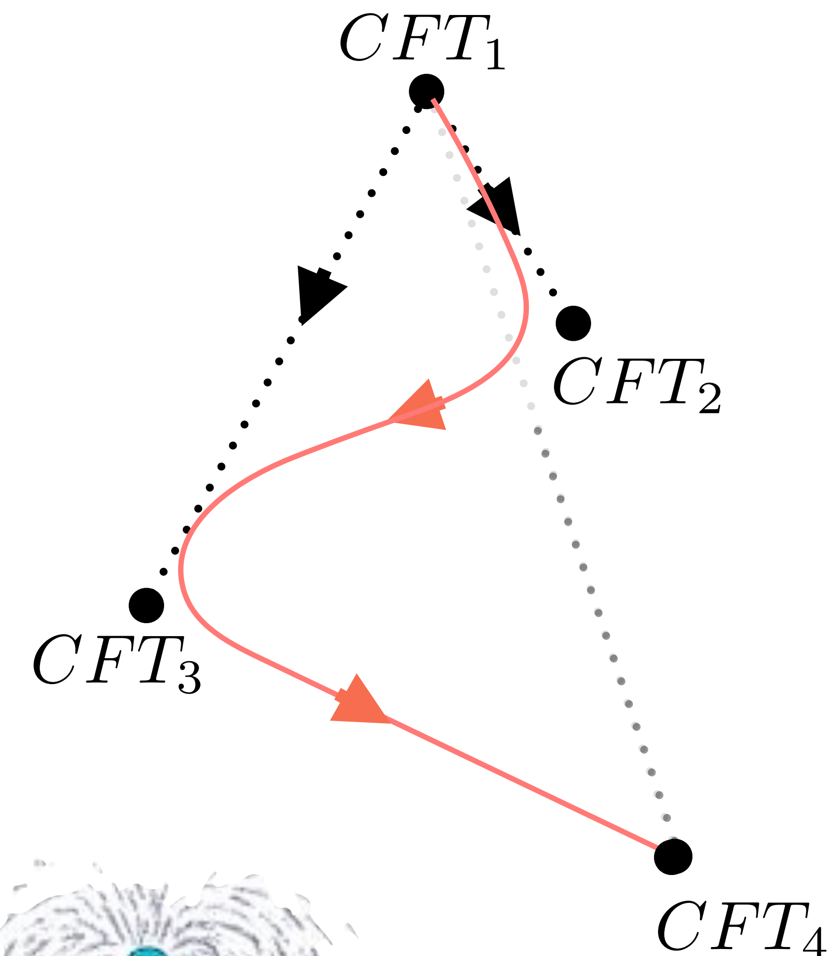
$$\pi \rho_{--}(q) = Im\left(\frac{32 i\pi q\left(\left(q^2+4\right)\tanh^{-1}\left(\frac{q}{2}\right) - 2q\right)}{4q^2(8\pi q - \lambda)(\lambda + 8\pi q) + \lambda \tanh^{-1}\left(\frac{q}{2}\right)\left(-128\pi q^3 + 4\lambda\left(q^2+4\right)q - \lambda\left(q^2-4\right)^2 \tanh^{-1}\left(\frac{q}{2}\right)\right)} \right)$$

$$\lambda = \frac{\pi}{k}$$



Delacretaz, ALF, Katz, Walters, Vitale

Conformal Bootstrap and QFT

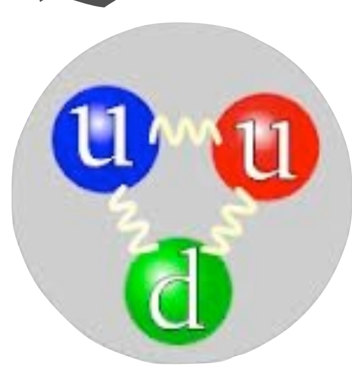


QFTs are on RG flow between (or nearly between) CFTs

E.g. The Standard Model

???

QCD



QED

$m_Z^{-1} \sim 10^{-18}\mathrm{m}$ $\quad\Lambda_{\mathrm{QCD}}^{-1} \sim 10^{-15}\mathrm{m}$ $\quad m_e^{-1} \sim 10^{-12} m$