Hierarchy of HIerarchies

Michael Dine

Department of Physics
University of California, Santa Cruz

Northeastern University, in Honor of Pran Nath, May, 2019

Princeton, 1982

Remember preprints? I won't forget the arrival of a preprint at IAS by Arnowitt, Chamseddine and Nath, explaining how to think about supersymmetry and its breaking within the framework of supergravity. It was not aligned with my thinking at that moment, and I was very skeptical. On point after point, I was sure that they could not be correct, and each time, after some thought, I realized they were. That paper changed my understanding not just of supergravity but of the role of effective field theory in understanding loow energy effective field theory.

From that time on, Pran has been a leader in the subject of supersymmetry, its phenomenology, and the study of its emergence in low energy effective field theories, and I want to acknowledge my debt to Pran today.

Locally Supersymmetric Grand Unification

A. H. Chamseddine, R. Arnowitt, and Pran Nath

Department of Physics, Northeastern University, Boston, Massachusetts 02115

(Received 12 July 1982)

A locally supersymmetric grand unification program is proposed which couples the $N=1$ supergravity multiplet to an arbitrary grand unified gauge group with any number of left-handed chiral multiplets and a gauge vector multiplet. A specific model is discussed where it is shown that not only do the gravitational interactions eliminate the degeneracy of the vacuum state encountered in global supersymmetry, but simultaneously they can break both supersymmetry and $SU(2)\otimes U(1)$ down to a residual $SU(3)^c\otimes U(1)$ symmetry at ~ 300 GeV.

PACS numbers: 12.10.En

Recently much interest has been devoted to supersymmetric grand unified theories.[1-5] All existing supersymmetric grand unified models are based on *global* supersymmetry. In such theories it is generally easy to break spontaneously the internal, e.g., SU(5), symmetry, but more difficult to break supersymmetry itself. In this paper we propose a new type of supersymmetric grand unified model based on *local* supersymmetry. We consider here $N=1$ supergravity[6] coupled to left-handed chiral scalar[7] and gauge multiplets.[8] We will see that the supergravity couplings automatically produce a spontaneous breaking which removes the degeneracy of the

The present moment: May, 2019

We have been obsessed with three hierarchy problems for several decades.

1. The weak scale hierarchy
2. The cosmological constant problem
3. The strong CP problem

There are other problems, which might fit in this list:

- Origin of the dark matter. Could be tied to one of the other problems (e.g. WIMPs in susy, axions for strong CP).
- Inflation: slow roll requires surprisingly light fields; possibly new, finely tuned scale; fine tuning to obtain sufficiently flat potential. Perhaps supersymmetry or compositeness?

Alternatively, these might introduce their own independent hierarchies.

We will not focus on these in this talk.

For each of these problems, proposed solutions:

1. Weak scale hierarchy: supersymmetry, dynamical electroweak symmetry breaking (technicolor, and variations), warped spaces, little Higgs, twin Higgs; dynamical explanations (“relaxion")
2. The strong CP problem: axions, $m_u = 0$, Spontaneous CP violation (Nelson-Barr mechanism)
3. Cosmological constant problem: anthropic explanation, others(?)

Hierarchical ordering of hierarchies:

Operator dimension. Lower dimension $\Rightarrow$ more finely tuned:

1. Cosmological constant: Dimension 0
2. Higgs mass: Dimension 2
3. $F\tilde{F}$: Dimension 4

Anthropic Hierarchy

Given that the anthropic principle has raised its ugly head, it's interesting to consider: which of these problems might most *plausibly* be solved by anthropic considerations:

Ranking of problems by their potential anthropic significance:

1. Cosmological constant: must be small (everything else fixed) to satisfy the most primitive conditions for intelligent observers (existence of structure in the universe)
2. Weak scale hierarchy: might be solved anthropically, perhaps by demand of details required for carbon-based life (e.g. stellar processes) (Dimopoulos et al: "atomic principle")
3. θ: Hard to see significant consequence for existence of observers. $\theta \sim 0.1$ little consequence for stellar processes, nuclear physics,...(Ubaldi). I will discuss a recent suggestion of Kaloper/Terning.

Experimental Access to Natural Solutions

Again, a hierarchy of how accessible these problems and their *natural* solutions might be to experimental test:

1. Cosmological constant: no compelling natural solution. Not clear what to search for.
2. Weak scale hierarchy: If susy, and if not significantly tuned, accessible to accelerators. At this point, much of parameter space excluded. Higgs mass troubling. In a simple-minded approach, points to scale of order 30 TeV or so. Probably have to live with significant tuning if SUSY plays any role. Alternatives to SUSY don't fare much better.
3. Strong CP: Challenging. If dark matter produced in simplest cosmology, may be accessible to ADMX, other experiments. Lighter axions: proposals for search strategies. ADMX; other proposed searches will gradually sweep out parts of the parameter space.

Outline and plan:

1. The biggest problem: the c.c. Landscape/anthropic solutions.
2. Challenges to the traditional view of the electroweak hierarchy. Frame mainly in terms of supersymmetry.
3. Supersymmetry in a landscape – arguments against.
4. Supersymmetry in a landscape – an argument for.
5. Strong CP: a challenge to the landscape program

The Cosmological Constant Problem

Within our hierarchy of hierarchies, the cosmological constant problem stands at the top. Because it involves contributions to the unit operator, it exhibits the most severe tuning. Because without (unbroken) supersymmetry, no known symmetry protects the cosmological constant, no (compelling) natural solution has been put forward.

With supersymmetry, vanishing cosmological constant results from supersymmetry if:

1. Supersymmetry is unbroken
2. There is an unbroken R symmetry.

In nature, supersymmetry is clearly badly broken. It is hard to see how supersymmetry can account naturally for a cosmological constant as small as we observe.

Anthropic solution of the cosmological constant: Weinberg with some updating

Suppose the underlying theory (string theory(?)) possesses a vast number of (metastable) ground states. Among these, the cosmological constant is a random variable. Other constants of nature also presumably vary (“scan"). Consider that set where all other constants are as we observe, but the c.c. varies. Somehow (cosmologically) the universe samples all of these “vacua".

Weinberg (1987): Most of these universes will not contain intelligent observers. A minimal condition for existence of observers: formation of structure (stars, galaxies). Requires that c.c. not dominate the energy density of the university until fluctuations become non-linear, about 1 billion years after the big bang. Translates into an *upper limit* on the c.c. about and order of magnitude larger than subsequently observed.

Subject to many criticisms but a *prediction* of the dark energy; yielded roughly the quantity observed.

Does string theory produce a landscape?

String theory is a framework in which the features of the low energy theory follow from a microscopic, “ultraviolet complete" framework. Various features emerge. In constructions we understand:

1. Gauge groups
2. Generations of quarks and leptons
3. Varying degrees of supersymmetry
4. Calculable (in principle) coupling constants, *determined by expectation values of massless dynamical fields (“moduli").*

Limitations – and a Landscape?

But:

1. Without supersymmetry, difficult to find non-supersymmetric vacua which can be analyzed in any systematic approximation
2. Indeed, hard to understand why supersymmetry not preferable.

Closing one's eyes to these problems, Bousso, Polchinski noted that there are many types of fluxes (analogous to electric and magnetic fluxes) in compactificaions of string theory. Each of theses fluxes can take on a range of values. Turning on these fluxes, if somehow the various moduli are stabilized (fixed), suggests the possibility of a vast array of ground states. E.g. N fluxes, taking m possible values: m^N states. Leads to *landscape* (Bousso, Polchinski; Susskind).

Simplified view

Imagine one has N types of fluxes, n_i, which can take m values. BP *Model*:

$$\Lambda = \sum_{n_i} \Lambda_i^2 n_i^2 - \Lambda_0. \tag{1}$$

Of order m^N states (take $m \sim N \sim 1000$, say). If Λ_i's comparable, random numbers, a nearly continuous distribution ("discretuum") of values of Λ. Expect uniform probability, $P(\Lambda)$, to find Λ near $\Lambda = 0$. States of similar, small, Λ have very different values of the n_i's.

KKLT put forth a more detailed which makes this more plausible, but the existence of a landscape remains, at best, conjectural. (Some theorists, e.g. Banks, Sethi, offer principled objections).

In any case, a plausible story to account for the c.c. from a structure like string theory.

But if this is the underlying picture, it is not only the c.c. which scans. There should be distributions of discrete features (gauge groups, matter content,...) and continuous parameters of low energy effective theory, such as the Higgs mass, and the gauge and Yukawa couplings.

Absent a derivation of a landscape from string theory, educated guesses (esp. Douglas-Denef) as to *statistics*.

Schematically, would like to know (for a given choice of discrete parameters) the distribution of continuous parameters of the low energy theory:

$$P(\Lambda, m_h^2, y_{ff'}, \alpha_i, \ldots)$$

Then might ask:

1. Do most of states compatible with some set of anthropic constraints exhibit certain features (axions? low energy supersymmetry?)
2. Do cosmological considerations favor some class of states?

Absent strong correlations among physical quantities, in such a picture, one might expect the low energy theory – its degrees of freedom and other parameters, to be either anthropically determined or random. There are features of the Standard Model which appear to be neither:

1. Multiple generations
2. Patterns in the CKM matrix and in the masses of quarks and leptons
3. Perhaps most dramatically, the smallness of θ

The Electroweak Hierarchy Problem

Having opened this Pandora's box – landscape plus anthropics – to understand the c.c., we need to revisit our other naturalness questions.

In our hierarchical ordering, having "solved" the c.c. problem, the next in line is the electroweak hierarchy. Not necessary to restate the problem here. But worth stressing that the issue is an operator of dimension two. So less severe by many orders of magnitude than the cc problem (solution of the larger may sweep away the smaller).

A variety of proposed solutions.

1. Technicolor/Randall Sundrum
2. Little Higgs/twin Higgs
3. Supersymmetry
4. Anthropic/landscape

The last is the most frightening. Precisely because the problem is less severe than the cc., it might be *even easier* to solve in a landscape. We will focus on (3) and (4).

Supersymmetry and Naturalness The Traditional Story

SUSY has long been the focus of intense interest. There are three reasons for this, but many of us are feeling growing unease:

1. Hierarchy problem: But SUSY now looks severely tuned
2. Dark matter: But simplest SUSY WIMP largely excluded
3. Unification: Always worried this might be an accident

In varying degrees, other proposals are under similar stress. Possibly we've just not been clever enough. Much effort, with, without supersymmetry looking for natural explanations, signals yet to be discovered. More on this in today's talks.

Strains on Natural Supersymmetry

There are two sources of tension:

1. Supersymmetry exclusions: typically greater than 1 TeV for colored particles; weaker constraints for color neutral states.
2. The Higgs mass: in simplest version, requires stop squark of order 30 TeV. With adjustment of parameters (A parameter), additional fields, can be better (e.g. Shadmi). But at generic points in the parameter space, the tuning is large, a part in 1000 or worse.

30 TeV as the SUSY breaking scale would correspond to a version of *split supersymmetry*. How much tuning is *too much*? Perhaps this sort of tuning, while very troubling for discovery, is not so shocking. Of course, for discovery of supersymmetry it may be problematic, unless some states are light (e.g. as in *split supersymmetry*).

The nightmare scenario

Like the cosmological constant, the Higgs mass is simply selected from a distribution by anthropic considerations. Principles which might require this have been given various names: “the atomic principle", and the “carbonic principle". These principles are subject to even sharper critiques than Weinberg’s. The Higgs mass is very sensitive to what one holds fixed (e.g. quark Yukawa’s).

But one can make a weak statement: if other quantities *are* held fixed, and the Higgs mass is allowed to vary, our existence is in jeopardy. So, without offering a detailed explanation, it is plausible that it is the result of such anthropic selection.

It is too early to despair. Perhaps we have just been a bit unlucky, and supersymmetry, or composite Higgs, or something else, is around the corner.

But it is also possible that, even if a landscape picture holds, there are correlations. Could it be, for example, that most of the states in a landscape with a light Higgs also have supersymmetry? If we thought about this carefully enough, could we decide what we expect for the realization of supersymmetry and the scale of its breaking?

Hierarchy of Puzzles in a Landscape

There are many puzzling questions about how a landscape might emerge from string theory and how it might look. We would like to know things like the numbers of theories with a given gauge group and low energy particle particle content, the distribution of couplings, and the like. But there are more primitive questions:

1. States with a great deal of supersymmetry ($N = 2, 4, 8$) arising from string theory are the only states we can really claim to understand. Do states without supersymmetry actually arise?
2. Among these, is four dimensions in some suitable sense common?
3. Starting with the assumption that the low energy theory is generally covariant in four dimensions, with broken supersymmetry and a small cosmological constant, is this system typically stable against decay to states with negative c.c.?
4. Among supersymmetric states in a landscape, is there some favoring of low or high scales of supersymmetry breaking?

I don't really have any idea how to address the first two questions, but will offer some thoughts on the questions of stability and the favoring of scales. Indeed, stability is one of the most dramatic, and perhaps tractable, puzzles of a landscape picture.

This question, in turn, has two components: classical stability, the existence of some large number of local minima of some underlying potential, and *quantum stability*, the requirement of a large number of very long-lived states.

I will argue that both point to a role for supersymmetry at scales well below some fundamental scale. The question which I won't resolve is: how well below?

SUSY in a Landscape

One can imagine organizing states in a landscape in terms of their degree of supersymmetry

Landscape branches:

1. Non supersymmetric states
2. States which are approximately supersymmetric “by accident"; one can think of this as tuning of parameters. (More precise shortly)
3. States exhibiting dynamical supersymmetry breaking: (More precise shortly)

Approximate Supersymmetry

How might approximate supersymmetry arise in a landscape?

Douglas and Denef considered the likelihood in classes of flux vacua, one had approximate supersymmetry, simply as a result of random choices of flux (we'll call this "tuned supersymmetry"). Found for low scale unlikely, roughly

$$P(F) \sim |F|^6 \tag{2}$$

Result understood in terms of a low energy theory with a light field (goldstino), with a uniform distribution of superpotential parameters (as complex numbers) (Z Sun, M.D.)

$$W = W_0 + \gamma Z + \mu Z^2 + \ldots \tag{3}$$

Low scale supersymmetry breaking requires the leading superpotential parameters all small. Assuming uniformly distributed (as complex numbers) accounts for Douglas-Denef statistics. So low supersymmetry rare as a random phenomenon. In this context, can't provide a *natural* explanation of the hierarchy: a light Higgs and a high scale of supersymmetry is far more likely than a light Higgs and low scale supersymmetry.

A different possibility: exponential separation of susy scale from the high scale (dynamical supersymmetry breaking). Supersymmetry a good symmetry at some high energy scale; In terms of explicit string models, supersymmetry unbroken “at tree level". Low energy theory breaks supersymmetry

$$F = Me^{-\frac{8\pi^2}{g^2(M)}} \tag{4}$$

In this case, if g^2 roughly uniformly distributed, roughly equal probability of susy breaking per decade. (Not necessarily a prediction of low energy supersymmetry breaking). This remains the case requiring (selecting for) small cosmological constant.

A fourth branch, suggestive of very low energy supersymmetry breaking

If $\langle W \rangle$ is dynamically determined, then states with small cosmological constant are most likely with low supersymmetry breaking scale. Discrete R symmetries could account for small W. But these seem likely to be rare in a flux landscape (Sun, M.D.).

Summary: Landscape Statistics

1. Non-supersymmetric states: There might be so many more non-susy than susy states that there are many more non-susy states with light Higgs than supersymmetric ones.
2. Accidental supersymmetry: The probability of a SUSY breaking with order parameter F goes as
$$P(F) \propto F^6 \tag{5}$$
3. Dynamical breaking of supersymmetry:
$$P(F) \propto \log(F) \tag{6}$$
Roughly equal distribution of states with scale. For a string theorist, such states would be states with unbroken supersymmetry at tree level.
4. Dynamical breaking of supersymmetry and R symmetry:
$$P(F) \sim \frac{1}{|F|^2} \tag{7}$$

The last two possibilities are realizations of the traditional picture of tuning and dynamical supersymmetry breaking. States with light Higgs might be at relatively low scales. But mightn't one expect that states states exhibiting in the sense of the third branch are exceptional (just as the tuned states are rare)?

It is here that the question of stability might provide a useful discriminator between the branches.

So we turn to a more serious investigation of stability, both classical and quantum.

Classical Stability

Various models have been studied to understand the likelihood of classical stability in a landscape. With N fields, we have, at any given stationary point of the potential, to diagonalize and $N \times N$ matrix. If we simply assume that each eigenvalue has a $1/2$ chance to be positive, stability occurs in 2^{-N} of the states. But if one truly models as a random matrix (McAlister et al) the suppression is far more severe, as e^{-cN^2} for some constant c. So classical stability is rare without further restrictions on the space of states.

Quantum Stability

Even if we find a suitable local minimum with a small cosmological constant, this state is invariably surrounded by an exponentially large number of negative c.c. states. It is crucial that decays to *all* of these states be suppressed.

E. Weinberg et al: a simple model with N scalar fields and a random quartic potential. Numerical studies indicated an exponential suppression of stability. Paban and M.D. understood analytically. Requiring that the smallest bounce action be larger than some fixed number, B_0, yields an exponential suppression with N. For example, for $N = 100$, requiring that B_0 give a lifetime longer than the age of the universe (not in our past last cone) gives a suppression of order 10^{-56}.

Supersymmetry: A guarantor of stability

Classical stability: Consider a low energy theory with spontaneous supersymmetry breaking (in supergravity context, means a light gravitino). Necessarily a light chiral or vector multiplet containing a gravitino. Classical stability a question of whether all masses for a small number of light states are positive. No exponential suppression.

[McAlister et al argued for e^{-N} suppression. But assumptions about couplings violated unitarity; study of string theories indicated results consistent with naive expectations]

Quantum stability

What might account for a high degree of stability?

1. Small string coupling(s)?
2. Large radius of compact spaces?

Claim: these seem unlikely to be particularly generic.

But another possibility appears robust: approximate supersymmetry.

Stability with Supersymmetry

With exact supersymmetry in flat space, the vacuum is stable. This can be understood as a consequence of the existence of *global* supercharges, obeying the familiar algebra:

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2P^\mu(\sigma_\mu)_{\alpha\dot{\beta}} \tag{8}$$

As a result, there are *no tachyons* and *no tunneling* (Weinberg, 1981).

Stability with slightly broken supersymmetry

Classical stability: With (slightly) broken supersymmetry, expect only a few states with masses of order $m_{3/2}$ potentially tachyonic. So don't expect a big suppression of stability (i.e. suppression as e^{-aN}).

Quantum stability: expect tunneling still vanishes or highly suppressed. Two classes of trajectories: directions with fields much more massive than $m_{3/2}$, and directions with masses of order $m_{3/2}$. For the former, for a broad class of models (Festuccia, Morisse, M.D.), one has a general formula:

$$\Gamma \propto e^{-2\pi^2\left(\frac{M_p^2}{m_{3/2}^2}\right)} \tag{9}$$

For the latter, anything is possible, but as for classical stability, only a few trajectories must be suppressed; don't expect e^{-aN} type suppression of stability.

So in a landscape, supersymmetry, broken at a scale well below M_p, might be common. But arguments for *TeV scale* supersymmetry are not strong. Perhaps dark matter, other questions might pin down the scale.

Naturalness – status

Naturalness is under stress. Possible resolutions:

1. We've been unlucky. There is new physics at TeV scale accounting for the electroweak hierarchy. Interesting challenges for model-builders, experiments.
2. Naturalness overthrown: landscape as a model. Cosmological constant/electroweak scale.
3. Naturalness not necessarily lost in a landscape: possible role for supersymmmetry. Stability. But may look tuned.
4. Landscape challenge: where does strong CP fit in? Other peculiar features of SM? *Principled reason for skepticism about landscape ideas*

Discovery possibilities and their implications

1. Evidence for natural solutions (susy, evidence for light pseudogoldstones,...)
2. WIMP dark matter or similar: evidence for new interactions
3. Further exclusions – further evidence for tuning
4. Shocking discoveries? Another Higgs doublet? Potential to overthrow both *naturalness* and the *landscape*. **Who Ordered That?** (At least the muon is naturally light!)