Acoustics and Fourier Transform

Physics 3600 - Advanced Physics Lab - Summer 2021
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I. INTRODUCTION

Time is fundamental in our everyday life in the 4-dimensional world. We see things move as a function of time. On the other hand, although sound waves are composed of moving atoms, their movement is too small and the frequency of the vibration is too fast for us to view directly. It is thus easier to describe sounds in *frequency* space rather than *time* space. We can transform sound, or many other things in physics for that matter, from *time* space to *frequency* space by the technique of Fourier transform (described in Appendix). In this experiment you will record time-varying sound wave patterns and transform them into frequency spectra (amplitude as a function of frequency).

Natural sounds are almost never pure sine waves having one single frequency. By Fourier transforming various sounds you will investigate your voice, beat frequencies, and harmonics in musical instruments. Sounds picked up by a microphone and preamplifier will be stored digitally in an oscilloscope, saved to a file, then analyzed with Fast Fourier Transform (FFT) software.

II. APPARATUS

microphone, preamplifier (Rainbow Labs AA-1), storage oscilloscope, speaker,
tuning forks, guitar, stringed instrument, electric piano, etc.
FFT software - FFT in Excel, FFT in Matlab, EasyPlot

III. PROCEDURE

A. Basic FFT

1. Plot the sine wave, y=sin(2pi100x) from x=0 to 0.1 seconds, using 2048 points.
   (1) Compute the FFT with a Hamming filter.
   (2) The FFT frequency axis must be rescaled by noting that the spacing between frequency points is equal to the reciprocal of the TOTAL time range Δt.

For this case, transform the FFT x-axis point spacing to $\delta f = 1/\Delta t = 1/0.1$ s = 10 Hz per point (x=x/0.1).
NOTE: do not use bar graphs for your plots, use points and connecting/fitted lines.

2. Plot the FFT only in the **region of interest** (ROI) near 100 Hz.
   a. What is the frequency f_o at the peak of the FFT? (Retransform if you don't get the correct value.)
   b. What is the FWHM (full-width at half-maximum) linewidth, Γ? What is full range of frequency Δf ?

3. Plot the sine wave with larger number of cycles, from x = 0 to 1.0 s. Compute and scale the FFT.

4. Plot both FFTs on the same graph (scale each to a peak maximum of 100 for comparison).
   a. What is the new Γ ? What is full range of frequency Δf?
   b. How many cycles are recorded for each of the two plots?
   c. How does the number of recorded cycles relate to the linewidths Γ?
   d. How does the number of recorded cycles relate to the full range of frequency Δf?
   e. High-precision FFT requires what number of collected cycles: (i) low (1-10) or (ii) high (>>100)?
   f. Large frequency range FFT requires what: (ii) short time collection or (ii) long time collection?

IMPORTANT - before performing any FFT, always zoom in the timescale to make sure that you can see periodic oscillations that have more than one point per cycle!

You must show the TA your plots at the end of each section before proceeding to the next section.

B. Sine and Square Waves

1. Set up the function generator and scope to record waveforms.
2. Set the scope vertical scale to produce a reasonable amplitude.

3. Generate and store a 1 kHz sine wave voltage V(time) using the function generator.
   - □ Remember to always truncate the V(t) points down from 2500 to 2048.
   - □ Apply the FFT with Hamming filter.
   - □ Transform the *f*-axis by x = x/Δt, where Δt is the total time for **2048 points**.

   a. Repeat the FFT for a 1 kHz square wave.
   b. Plot and discuss the **differences** in the FFTs for the sine and square waves.
   c. Connect the function generator to a speaker. Discuss what you hear from the 2 waves.
   d. What is unusual about the square frequency wave spectrum?

C. Tuning Forks

1. Select two tuning forks having the same pitch.
2. Observe the beating of both forks vibrating simultaneously using a time base of **~0.5 sec/square.**
   a. Plot and discuss the waveform, but **do not** compute the FFT.
   b. What is the frequency of your beats and discuss their origin?

D. Whistle

Record and store many (~100) cycles of a whistle sound using a time base of ~0.025 sec/square.
   a. Plot the FFT to look for the fundamental and any harmonics.
   b. Discuss what the FFT looks like.
   c. How distinguishable are whistle sounds from two different people whistling with the same pitch and intensity?
   d. What is the physiology of whistling?

E. Human Voice

Observe a vowel sound (*a,e,i,o,u*) on the scope.
   a. Record and store one vowel sound.
   b. Compute the FFT and look for harmonics far above the fundamental frequency.
   c. Can you relate the quality of the voice sounds that you hear to the FFTs?

F. Musical Harmonics

Here you will explore the difference in the sound produced by two different musical instruments playing the same note. This difference is related to musical "timbre," usually pronounced "tamber" as in the first syllable of the word tambourine.

Notes on Timbre - Why do different instruments sound very different when they produce the same note or pitch. The note A-440 played on a guitar sounds much different than the same note played on a trumpet. This property is referred to as timbre or tone color. Musical instruments have acoustical properties determined by the construction materials and especially their shape, which determines their resonant character. The main difference in the sound comes from the set of *harmonics* (multiples of the fundamental frequency). Most instruments don't produce a single frequency, instead, they produce acoustic vibrations at the fundamental frequency, f_o, and also at the harmonics, $2f_o$, $3f_o$, $4f_o$, etc. Thus, one instrument may produce high amplitudes at $3f_o$ and $5f_o$, while another instrument may produce high amplitudes at $6f_o$, $7f_o$, $8f_o$, and $9f_o$. In addition, the amplitudes of the harmonics will vary in time differently for different instruments.

Using the guitar and another instrument (flute, single string, electric piano, etc.), observe the same note (approximate frequency) on the scope.

Record and store the waveforms of the guitar and at least one other instrument.

a. Discuss the **differences** in their FFTs.
b. Does the guitar have more than one fundamental?
c. Make a single **plot** of the relative amplitudes of the harmonics for the two instruments.
   Plot amplitude A versus the number N, where $f = Nf_o$, and N = 1, 2, 3 ,...
   Scale the most intense peak of each instrument to 100.
d. Discuss the A(N) plot of the relative amplitudes.

G. Optional

Plot the sine wave, y = sin(2pi100x) from x = 0 to 0.1 seconds, using 100 points and only 10 points.
Does the apparent frequency change? Explain.

Discuss the *Nyquist frequency*.
(This can be confusing, so simply consider the minimum number of points you need per cycle.)

Bring in a musical instrument or other noise-maker and measure its harmonic spectrum.

IV. APPENDIX A: FOURIER TRANSFORM

A. Introduction

The Fourier transform (FT) is one type of mathematical transformation that changes or maps one axis variable to another variable. It is widely used for transforming a *time*-varying waveform A(t) into a *frequency*-varying spectrum B(f). An example we will investigate in this lab is the transformation of sound waves. Sound waves traveling in a medium (solid, liquid, gas) are composed of time-varying oscillations of the density of the atoms or molecules. They are longitudinal pressure waves. When characterizing a particular sound it is more convenient to study its *frequency* spectrum of the amplitude, B(f), which is the Fourier transform of the time-varying amplitude, A(t).

Consider the example of a pure *monochromatic* sound wave of frequency f_o, where the amplitude is a simple sine wave, A(t) = $A_o \sin(2\pi f_o t)$. We could also have written the sine function as $\sin(\omega_o t)$, where $\omega_o = 2\pi f_o$. Note that the argument of the trigonometric function $\omega_o t$ must be dimensionless, so ω must have units of s^{-1} (radians/s), whereas the units of frequency f are Hertz (cycles/sec). The Fourier transform of A(t) has a narrow peak at f_o. In other words, B(f) only has nonzero amplitude near the frequency f_o.

B. Basic Theory

There are two forms of the FT, discrete and integral. The integral form of the FT is given by

$$A(t) = \int_{-\infty}^{\infty} B(f) e^{i2\pi f \cdot t} df$$

$$B(f) = \int_{-\infty}^{\infty} A(t) e^{-i2\pi f \cdot t} dt$$

C. Fast Fourier Transform

Software programs use a type of algorithm referred to as Fast Fourier Transform (FFT) for computations. FFT software routines require that the number of points in the original curve be equal to $N = 2^n$, such as N = 256, 512, 1024, 2048, etc. After generating the FFT graph from the sine wave y = sin(2pi100x), the frequency axis must be rescaled. This is done by converting the point-to-point frequency interval

$$\delta f = 1 / \Delta t,$$

where Δt is the total range of the time axis. To produce a finer frequency scale a larger Δt must be used.

Note that the total FFT yields two peaks because the f-axis has a mirror image about the midpoint of the f-axis. Generally only the first half of the f-axis is used and the frequency is relative to f=0 on the left. The exception is when there are less than two points per cycle, at which point the FFT two peaks move towards the center of the f-axis and cross each other. In this case, the correct FFT peak is on right half of the f-axis, but its frequency is still relative to the left-hand origin. You can test this yourself.

D. Examples

Let's look at a few FT examples, for a damped sine wave and a Gaussian function.

D.1. Damped Sine Wave

Consider the damped sine wave of frequency f_o for $t > 0$ given by

$$A(t) = A_o \sin(2\pi f_o\, t)\, \exp(-t / \tau).$$

The sine wave has an initial amplitude A_o, but decreases exponentially in time with the damping time constant tau τ. As before, the argument of an exponential must be dimensionless so the units of t and τ are time. When the damping is comparatively small ($2\pi f_o t >> 1$, or $t << 1/2\pi f_o$) the FT of A(t) is a Lorentzian function given by

$$B(f) = \frac{A_o}{2}\sqrt{\frac{1}{(f - f_o)^2 + (\Gamma/2)^2}}.$$

Another generalized form for the Lorentzian is $\boxed{B(f) = \dfrac{B_o(\Gamma/2)^2}{(f - f_o)^2 + (\Gamma/2)^2}.}$

This form is ***normalized*** such that the maximum is B_o at $f = f_o$.

The Lorentzian function has a "bell shape" with a maximum of B_o at $f = f_o$. The full-width at half-maximum (FWHM) linewidth is Γ in units of Hz. The damping parameter is related to the time decay by $\Gamma = 1/\pi\tau$. Small damping means $\Gamma << f_o$, or large Q-factor $Q = f_o / \Gamma$.

D.2. Gaussian

Next, consider the Gaussian function of time given by

$$A(t) = A_o \exp\left[-2.77\, (t - t_o)^2 / \tau^2\right].$$

The Gaussian function is also bell shaped, having a normalized maximum amplitude A_o at $t = t_o$. The factor 2.77 makes τ the FWHM, which has units of time. The FT of A(t) is another Gaussian function given by

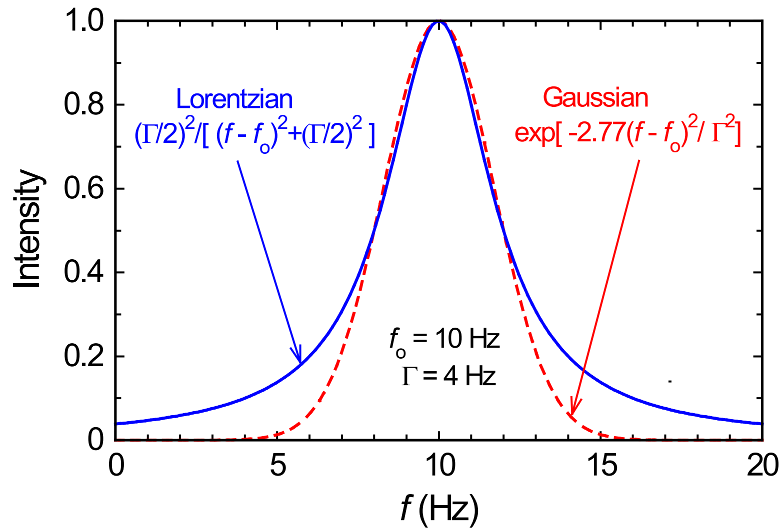
$$B(f) = B_o \exp\left[-2.77\, f^2 / \Gamma^2\right],$$

which has its maximum at $f = 0$ and a FWHM of Γ, both in units of Hz. The analogous shifted form at f_o is

$$\boxed{B(f) = B_o \exp\left[-2.77\, (f - f_o)^2 / \Gamma^2\right].}$$

D.3. Comparison of Lorentzian and Gaussian

The graph shows the difference between the Lorentzian (solid curve) and Gaussian (dashed curve) functions, with each having a central position at f_o = 10 Hz and a FWHM of Γ = 4 Hz. The Lorentzian has broader tails and a slightly sharper peak.



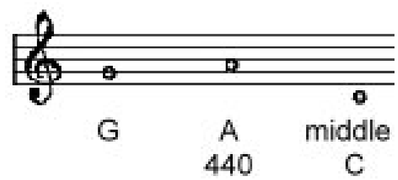
Lorentzian functions usually describe physical systems in which there is a characteristic *relaxation time* τ. This often happens to systems that are pushed out of equilibrium and relax back to equilibrium with a characteristic relaxation time. An example would be a system in thermal equilibrium that is perturbed by a temporary heating pulse.

On the other hand, Gaussian functions usually describe *inhomogeneous* physical systems that have a distribution in one of their physical properties. This often occurs in optical spectra. An example is the spectra of light coming from a fluorescent room light. Ideally, excited atoms emit light at a precise wavelength, giving rise to light emission in a very narrow spectral range of wavelength. However, the excited atoms in the fluorescent tube's plasma are extremely hot and have a distribution of velocities. These atoms traveling at high speeds emit light wavelengths that are Doppler shifted in frequency, which leads to Gaussian broadening in the wavelength of the spectral emission.

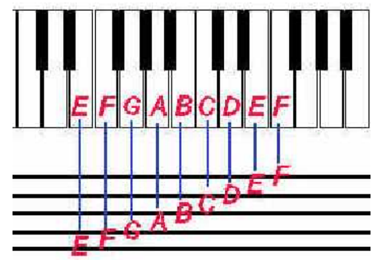
V. APPENDIX B: NOTES ON MUSIC THEORY

See http://www.teoria.com/, https://www.8notes.com/theory/

A. Musical Staff



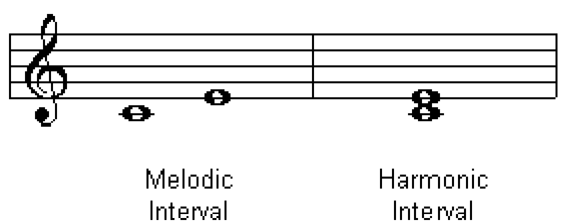
The staff on the right is a "treble clef" or "G clef," and the center of the sworl (second line from bottom) is the note G. The note below the staff is called, "middle C," but the frequency scale is usually set to the A above middle-C, normally at 440 cycles/second or Hertz. The frequency of a note is also call the "pitch" of the note.



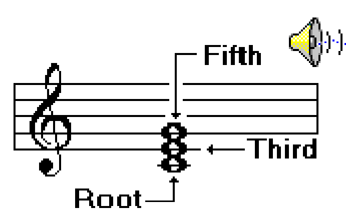
Starting from the bottom line and having single notes between the lines, the scale goes up: E, F, G, A, B, C, D, E, F. The two higher E and F notes at the top of the staff are one "octave" higher than the lower notes of the same letter, and consequently their frequencies are twice that of the lower notes.

B. Melodic and Harmonic Intervals

In a "melodic interval" the notes are played in succession. In a "*harmonic interval*" both (or several) notes are sounded simultaneously. The two notes here are C and E. The lower note is middle-C.



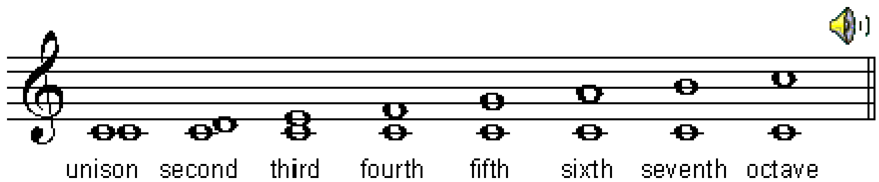
C. Musical Chord



Three or more notes sounded simultaneously form a chord. Traditionally, chords have been built by superimposing two or more thirds. For example, notes C-E-G form a chord or major triad. The note upon which the chord is founded is called the root. The other notes are called by the name of the interval they form in relation to the root name. This cord is the C-cord.

D. Numerical size of intervals

By counting the number of steps in an interval we obtain its *numerical size*. For example, a “fifth” is found by going from C to G (C, D, E, F, G). In this figure you can see the relationship between the notes and the numerical size of intervals:



The frequency relation in an interval is given by the ratio X:Y, so a fifth has the frequency ratio 3:2. An octave has the ratio 2:1. However, not all intervals of the same numerical classification are of the same size. That is why we need to specify the quality by finding the exact number of whole and half steps in the interval.

E. Mathematics Relation of Intervals

The A above middle C has a frequency of 440 Hertz. The A that is one octave higher has a frequency of 880 Hz, exactly the double the frequency. The mathematical relation is 880:440 or 2:1. The table shows the mathematical relation of several intervals, ordered from consonant (top) to dissonant (bottom).

Relation	Interval
2:1	Octave
3:2	Fifth
4:3	Fourth
5:4	Major Third
6:5	Minor Third
9:8	Major Second
16:15	Minor Second

F. Consonance and Dissonance

Intervals can be classified as consonant or dissonant according to the complexity of the mathematical relation between the note frequencies. This concept has changed during musical history and even today not all theoreticians agree. However, the classification in the table is quite useful.

Consonance	**Dissonance**
Unison	
Major and minor third	Seconds
Perfect fourth (considered a dissonance in harmony and counterpoint)	Sevenths
Perfect fifth	Augmented fourth
Major and minor sixth	Diminished fifth
Perfect octaves	