Definition 1  An integer $x$ is even if $x = 2k$ for some integer.

Definition 2  An integer $x$ is odd if $x = 2k + 1$ for some integer $k$.

Definition 3  An integer $a$ is divisible by an integer $b$ or $b$ divides $a$, denoted $b|a$, if there is an integer $c$ such that $bc = a$.

Definition 4  An integer $p$ is prime if $p > 1$ and the only positive divisors of $p$ are 1 and $p$.

Definition 5  An integer is composite if there is an integer $b$ such that $b|a$ and $1 < b < a$.

Definition 6  Set $A$ is a subset of set $B$ ($A \subseteq B$) if every element of $A$ is also an element of $B$.

Definition 7  Two sets $A$ and $B$ are equal if $A \subseteq B$ and $B \subseteq A$.

Definition 8  The intersection of sets $A$ and $B$ is $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

Definition 9  The union of sets $A$ and $B$ is $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

Definition 10  Let $A$ be a set. The power set of $A$, denoted $2^A$, is the set of all subsets of $A$.

Definition 11  The difference of sets $A$ and $A - B = \{x : x \in A \text{ and } x \notin B\}$.

Definition 12  The symmetric difference of sets $A$ and $B$ is $A \Delta B = (A - B) \cup (B - A)$.

Definition 13  The Cartesian product of sets $A$ and $B$ is $A \times B = \{(a, b) : a \in A, b \in B\}$.

Definition 14  $R$ is a relation on a set $A$ if $R \subseteq A \times A$. Notation: $(x, y) \in R$ is equivalent to $xRy$.

Definition 15  The inverse of relation $R$ is $R^{-1} = \{(x, y) : (y, x) \in R\}$.

Definition 16  Let $R$ be a relation on set $A$.

- $R$ is reflexive if $xRx$ for all $x \in A$.
- $R$ is irreflexive if $x \not\in R(x)$ for all $x \in A$.
- $R$ is symmetric if $xRy \rightarrow yRx$ for all $x, y \in A$.
- $R$ is antisymmetric if $(xRy \land yRx) \rightarrow x = y$ for all $x, y \in A$.
- $R$ is transitive if $(xRy \land yRz) \rightarrow xRz$ for all $x, y, z \in A$.

Definition 17  A relation $R$ on $A$ is an equivalence relation if $R$ is reflexive, symmetric, and transitive.

Definition 18  Let $n$ be a positive integer. Then the “congruence modulo $n$” relation on $\mathbb{Z}$ is defined as follows: $x \equiv y \pmod{n}$ if $n|(x - y)$.

Definition 19  Let $R$ be an equivalence relation on a set $A$ and let $a \in A$. The equivalence class of $a$ is $[a] = \{x \in A : xRa\}$.

Definition 20  $n! = n(n - 1) \cdot 3(2)(1)$, $0! = 1$

Definition 21  $(n)_k = n(n - 1) \cdots (n - k + 1)$

$$\binom{n}{k} = \frac{n!}{(n - k)!}$$

Definition 22  $\binom{n}{k}$ is the number of $k$-element subsets of an $n$-element set

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Definition 23  Let $f$ be a relation from set $A$ to set $B$. Then $f$ is a function from $A$ to $B$, $f : A \rightarrow B$, if

- the set of all possible first elements of $f$, called the domain of $f$, is $A$
- $(x, y) \in f$ and $(x, z) \in f$ imply $y = z$.

Notation: $(x, y) \in f$ is equivalent to $y = f(x)$.

The image of $f$ is the set $\text{im } f = \{y \in B : (x, y) \text{ for some } x \in A\}$.

Definition 24  Let $A$ and $B$ be sets, and $f : A \rightarrow B$. Then

- $f$ is one-to-one if $f(x) = f(y)$ implies $x = y$.
- $f$ is onto if for each $b \in B$, there exists an $a \in A$ such that $f(a) = b$.
- $f$ is a bijection if it is one-to-one and onto.

Definition 25  Let $A$, $B$, and $C$ be sets, and $f : A \rightarrow B$ and $g : B \rightarrow C$. Then $g \circ f$ is a function from $A$ to $C$ and $(g \circ f)(a) = g(f(a))$.

Basic Proposition  An integer is either odd or even but not both.