2. Since $f$ is a polynomial, it is continuous. Also since $f(0) = 1$ and $f(1) = 4$, and $1 < 2 < 4$, the Intermediate Value Theorem says that there must exist a real number $c$ in the interval $(0, 1)$ with $f(c) = 2$.

3. (a) $f(3) - f(1) = 8$.  
(b) $f(2) - f(1) = 6$.  
(c) $f(1.1) - f(1) = 4.2$.

4. (a) $f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{(x + h)^2 + 1 - x^2 + 1}{h} = \lim_{h \to 0} \frac{2x + h^2}{h} = \lim_{h \to 0} (2x + h) = 2x$.  
(b) $s'(1) = \frac{\sqrt{3(1 + h) + 1} - 2}{h} = \lim_{h \to 0} \frac{\sqrt{3h + 4 + 2} - \sqrt{3h + 4 + 2}}{h} = \lim_{h \to 0} \frac{3}{\sqrt{3h + 4 + 2}} = \frac{3}{\sqrt{4}} = \frac{3}{2}$.

5. (a) $6x^2 + \frac{1}{2} x^{-1/2} + 4(-\frac{4}{3} x^{-7/3})$.  
(b) $56x^5 \cos(x) \sqrt{1 - x^2} - x^5 \sin(x) \sqrt{1 - x^2} + \frac{1}{2} (1 - x^2)^{-1} \cdot (-2x)$.  
(c) $3x^2 \cdot 2x + x^3 \cdot 2 \cdot \ln(2)$.  
(d) $\frac{\cos(4y) - \ln(y)^2}{[3 - \tan^{-1}(y)]^{1/2}} \cdot \frac{1}{x^2 + 1}$.  
(e) $f'(x) = \frac{1}{x^2 + 1}$, $f''(x) = \frac{-2x}{(x^2 + 1)^2}$.

6. (a) $\ln(f) = \frac{1}{2} \cos(x) \cdot \ln(\sin(x))$ so $f' = \sqrt{\sin(x)} \cos(x) \left[ -\frac{1}{2} \sin(x) \ln(\sin(x)) + \frac{1}{2} \cos(x) \cdot \frac{\sin(x)}{\cos(x)} \right]$.  
(b) $\ln(g) = 4 \ln(x) - 8 \ln(x^2 - 2x + 5) - \frac{1}{3} \ln(5x + 1)$ so $h' = \frac{x^4}{(x^2 - 2x + 5)^8} \cdot \frac{\sqrt{5x + 1}}{5x + 1}$.  
(c) $\ln(h) = -\cos(x) + 19 \ln(x^3 + 3) - 2 \ln(\tan^{-1}(x))$, so $g' = \frac{e^{-\cos(x)} \cdot (x^3 + 3)^{19}}{(\tan^{-1}(x))^2} \cdot [\sin(x) + 19 \cdot \frac{3x^2}{x^3 + 5} - 2 \cdot \frac{1}{1 + x^2}]$.

7. Since $y' = 6 \cos(3x)$, point is $(\pi/4, \sqrt{2})$ and slope is $y'(\pi/4) = -3\sqrt{2}$, so equation is $y - \sqrt{2} = -3\sqrt{2}(x - \pi/4)$.

8. (a) Differentiating yields $2xy + x^2 y' + 2 \cdot 5x^4 y^6 + 2x^5 \cdot 6y^5 y' = 0$ so $y' = \frac{dy}{dx} = \frac{-2x y + 10x^4 y^6}{2x + 12x^5}$.

9. Differentiating, $100x^4 - (y^4 + x \cdot 4y^4 y') + e^{2x-y}(2-y') = 0$. Hence $y' = \frac{100x^4 - y^4 + 2e^{2x-y}}{4xy^3 + e^{2x-y}}$. At $(1, 2)$, $y' = \frac{-86}{33}$.

10. (a) Velocity is $(x', y') = (6e^{2t}, 6e^{6t})$, speed is $\sqrt{(6e^{2t})^2 + (6e^{6t})^2}$, acceleration is $(x'', y'') = (12e^{2t}, 36e^{6t})$.

11. (f) Since $f(1) = 10, f'(1) = 30$, linearization is $L(x) = 10 + 30(x - 1)$.

12. (a) Linearize $g(x) = e^{x}$ at $x = 0$ yields $L(x) = 1 + x$. Estimate is $L(0.04) = 1.04$.

13. Since $f(\pi) = 4\pi$ we see $g(4\pi) = f^{-1}(4\pi) = \pi$. Note $f'(x) = 4 + 3 \cos(3x)$.

14. (a) This is $f'(1)g(1) + f(1)g'(1) = 70$.  
(b) This is $f'(1)g(1) + f(1)g'(1) = -2/5$.  
(c) This is $f'(1)g(1) + f(1)g'(1) = f'(5) \cdot 8 = 16$.  
(d) This is $f'(f(1)) \cdot f'(f(1)) \cdot f'(f(1)) = 3 \cdot 2 \cdot 6 = 36$.

15. (a) $c = -1$ and $d$ is arbitrary.  
(b) $c = -1$ and $d = 3/2$. 

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