Our primary reference are the books of Costello and Gwillam [CG17, CG].

1. Prefactorization algebras

1.1. Definitions. A prefactorization algebra $\mathcal{F}$ on a topological space $M$, with values in $\text{Vect}^\otimes$ (the symmetric monodical category of vector spaces), is an assignment of a vector space $\mathcal{F}(U)$ for each open set $U \subseteq M$ together with the following data:

- For an inclusion $U \to V$, a map $\mu^V_U : \mathcal{F}(U) \to \mathcal{F}(V)$.
- For a finite collection of disjoint opens $\sqcup_{i \in I} U_i \subset V$, an $\Sigma_{|I|}$-equivariant map $\mu^V_{\{U_i\}} : \otimes_{i \in I} \mathcal{F}(U_i) \to \mathcal{F}(V)$

The maps $\mu^V_{\{U_i\}}$ are subject to following compatibility for sequences of inclusions:

- For a collection of disjoint opens $V_j \subset W$, and collections of disjoint opens $U_{j,i} \subset V_j$, the composite maps

\[
\otimes_j \otimes_i \mathcal{F}(U_{j,i}) \xrightarrow{\mathcal{F}(W)} \otimes_j \mathcal{F}(V_j)
\]

agree.

For example, the composition maps for the following configuration of three open sets are given by:
Note that $\mathcal{F}(\emptyset)$ must be a commutative algebra, and the map $\emptyset \to U$ for any open $U$, turns $\mathcal{F}(U)$ into a pointed vector space.

A prefactorization algebra is called \textit{multiplicative} if

$$\otimes_i \mathcal{F}(U_i) \cong \mathcal{F}(U_1 \cdots \cdots U_n)$$

via the natural map $\mu_{\{U_i\}}$.

1.1.1. \textit{An equivalent definition}. One can reformulate the definition of a factorization algebra in the following way. It uses the following definition.

A \textit{pseudo-tensor category} is a collection of objects $\mathcal{M}$ together with $\Sigma_{|I|}$-equivariant vector spaces $\mathcal{M}(\{A_i\}_{i \in I}|B)$ for each finite open set $I$ and objects $\{A_i, B\}$ in $\mathcal{M}$ satisfying certain associativity, equivariance, and unital axioms. Often times, pseudo-tensor categories are referred to as \textit{colored operads}, where the colors correspond to the objects in $\mathcal{M}$.

Every linear symmetric monoidal category $\mathcal{C}^{\otimes}$ determines a pseudo-tensor category $\mathcal{M}_C$ via the rule

$$\mathcal{M}_C(\{A_i\}_{i \in I}|B) = \text{Hom}_C(\otimes A_i, B).$$

Conversely, given a pseudo-tensor category $\mathcal{M}$, there is a “universal symmetric monoidal category” $\mathcal{S}\mathcal{M}$ whose objects consist of formal expressions

$$A_1 \otimes \cdots \otimes A_n.$$ The tensor product is defined in the obvious way. The hom spaces are defined by the formula

$$\text{Hom}_{\mathcal{S}\mathcal{M}}(A_1 \otimes \cdots \otimes A_n, B) = \mathcal{M}(\{A_1, \ldots, A_n\}|B).$$

Our favorite example of a pseudo-tensor category is associated to any manifold $M$. The pseudo-tensor category $\text{Disj}_M$ of \textit{disjoint open sets} in $M$ is given by the set of all connected open subsets of $M$. For every such finite collections of opens $\{U_i\}_{i \in I}$ and open $V$ one defines

$$\text{Disj}_M(\{U_i\}|V)$$

by the following rules:

- if the collection $\{U_i\}$ is pairwise disjoint and are all contained $V$, then $\text{Disj}_M(\{U_i\}|V)$ is the set with one element;
Composition in this pseudo-tensor category is defined in the obvious way.

One can check the following:

**Proposition 1.1.** A prefactorization algebra is a symmetric monoidal functor

\[ \mathcal{F} : S \text{Disj}_M \to \text{Vect}. \]

This motivates the following, more general definition. If \( C^{\otimes} \) is any symmetric monoidal category, we can consider “prefactorization algebras with values in \( C^{\otimes} \). Precisely, a prefactorization algebra on \( M \) with values in \( C^{\otimes} \) is a symmetric monoidal functor

\[ \mathcal{F} : S \text{Disj}_M \to C^{\otimes}. \]

Typically, for us, \( C^{\otimes} \) will be the symmetric monoidal category of vector spaces \( \text{Vect}^{\otimes} \), chain complexes \( \text{Ch}^{\otimes} \), or slight enhancements that we will discuss later on.

**Remark 1.2.**

1. Note that \( \emptyset \) is a connected open subset. Thus, for each prefactorization algebra we have an object \( \mathcal{F}(\emptyset) \in C \). According to the definitions, \( \mathcal{F}(\emptyset) \) is a commutative algebra object in \( C \).

2. One says that \( \mathcal{F} \) is *unital* if \( \mathcal{F}(\emptyset) \) is a unital commutative algebra.

3. In the language of colored operads, a prefactorization algebra is an algebra over the colored operad of disjoint open sets in \( M \), \( \text{Disj}_M \).

4. The category of prefactorization algebras themselves form a pseudo-tensor category \( \text{PreFact}_M \). If \( \{ \mathcal{F}_i \} \) is a finite collection of prefactorization algebras on \( M \), and \( \mathcal{G} \) is another prefactorization algebra then one defines

\[ \text{PreFact}_M(\{ \mathcal{F}_i \}|\mathcal{G}) = \text{Hom}(\otimes \mathcal{F}_i, \mathcal{G}). \]

Here, the tensor product is defined by \( (\otimes \mathcal{F}_i)(U) = \otimes \mathcal{F}_i(U) \). A homomorphism of prefactorization algebras \( \phi : \mathcal{F} \to \mathcal{G} \) is defined by the data: for each \( U \subset M \) open, a map \( \phi_U : \mathcal{F}(U) \to \mathcal{G}(U) \) subject to obvious compatibilities with the structure multiplication maps.

### 1.2. Examples.

#### 1.2.1. Associative Algebras.

The simplest examples of prefactorization algebras we give are on \( \mathbb{R} \), and are built out of Associative algebras. There is a map

\[ \text{AssocAlg} \to \text{PreFact}(\mathbb{R}) : A \to A^{\text{fact}} \]

The multiplicative prefactorization algebra \( A^{\text{fact}} \) assigns a copy of the associative algebra \( A \) to each open interval, \( A^{\text{fact}}((a, b)) = A \). The
structure maps are all given by the appropriate multiplication maps in the algebra. For example

\[ a \otimes b \otimes c \quad \varepsilon \quad A \otimes A \otimes A \]

\[ \downarrow \quad \downarrow \]

\[ ab \otimes c \quad \varepsilon \quad A \otimes A \]

\[ \downarrow \quad \downarrow \]

\[ abc \quad \varepsilon \quad A \]

The algebras we construct this way are \textit{locally constant}, meaning that the maps \( \mathcal{F}((a, b)) \to \mathcal{F}((c, d)) \) are isomorphisms for an inclusion of intervals \((a, b) \subset (c, d)\). In fact, any locally constant prefactorization algebra on \( \mathbb{R} \) recovers an associative algebra \( A_F = \mathcal{F}(\mathbb{R}) \), as the local constant property guarantees that \( \mathcal{F}((a, b)) \to \mathcal{F}(\mathbb{R}) \) is an isomorphism for each interval. In this way, we can produced an equivalence of categories

\[
\text{AssocAlg} \cong \text{PreFact}_{l.c.}(\mathbb{R})
\]

Here we still taking prefactorization algebras valued in vector spaces.

1.2.2. \textit{Bimodules as domain walls (defects).} In quantum field theories, we often consider \textit{defects}. These are a certain class of operators that are attached to submanifolds \( N \subset M \). For codimension 1 defects, known as domain walls, the operator acts to separate two different QFTs on either side of the wall. We expect to find that the domain wall is a bimodule for the algebra of local observables on both sides, in a compatible way.

This setup is very naturally described by bimodules for associative algebras. Let \( A, B \) be associative algebras. For a point \( p \in \mathbb{R} \) and \( M \) and \( A - B \) bimodule, we construct a prefactorization algebra \( \mathcal{F}_{A, M, B} \) on \( \mathbb{R} \) as follows.

- For an interval \( (a_1, a_2) \) with \( a_1 < a_2 < p \), we assign

\[
\mathcal{F}_{A, M, B}((a_1, a_2)) = A.
\]

- For an interval \( (b_1, b_2) \) with \( p < b_1 < b_2 \), we assign

\[
\mathcal{F}_{A, M, B}((b_1, b_2)) = B.
\]

- For an interval \( (a, b) \) with \( a < p < b \), we assign

\[
\mathcal{F}_{A, M, B}((a, b)) = M.
\]

For the structure maps, we have the module action \( a \otimes m \to am \), etc.
The last structure map we need to define is for the map $\emptyset \to (a, b)$, where $p \in (a, b)$. For this, there is no canonical choice, and we must prescribe an element $m_{(a,b)} \in M$ to each such interval.

Similarly, we may consider two defects: an $A-B$ bimodule $M$ attached at $p \in \mathbb{R}$, and a $B-C$ bimodule $N$ at $q \in \mathbb{R}$. We then can construct a factorization algebra $\mathcal{F}_{A,M,B,N,C}$ with this data, where the only new piece of information we need to prescribe is for intervals $(a, c)$ that contain both $p$ and $q$. The structure maps tell us that $V = \mathcal{F}_{A,M,B,N,C}((a, c))$ must receive maps from $M \otimes N$. Furthermore this map must factor through the internal $B$ action, and thus there is a map $M \otimes_B N \to V$, but this map may not be surjective. We come back to this construction in the last section of these notes.

In physics language, this is known as fusion of domain walls.

1.2.3. Enveloping Algebras. For more interesting examples, we consider factorization algebras on $\mathbb{R}$ with values in in the category $dg\text{Vect}^\otimes$ of differential graded vector spaces, where we invert weak equivalences.

Recall the definition of the Chevally-Eilenberg chain complex, for a Lie algebra $\mathfrak{h}$. This is a model for Lie algebra homology. Define

\begin{equation}
C_*(\mathfrak{h}) := \text{Sym}(\mathfrak{h}[1]) = \oplus_{n \geq 0} \wedge^n \mathfrak{h}[n]
\end{equation}

equipped with the following degree 1 differential

\begin{equation}
d_{CE} := [\cdot, \cdot] : \wedge^2 \mathfrak{h}[2] \to \wedge^1 \mathfrak{h}[1]
\end{equation}

extended to all of $\oplus_{n \geq 0} \wedge^n \mathfrak{h}[n]$ as a derivation. The Jacobi identity for the bracket ensures that $d_{CE}^2 = 0$. From this, we can show that $C_*(\mathfrak{h}) \otimes_{\mathbb{K}} U\mathfrak{h}$ is a free resolution of $\mathbb{K}$ as a $U\mathfrak{h}$ module, where the differential is

\begin{equation}
h_1 \wedge \cdots \wedge h_n \otimes x \mapsto d_{CE}(h_1 \wedge \cdots \wedge h_n) \otimes x + \sum (-1)^{i-n} h_1 \wedge \cdots \wedge \hat{h}_i \wedge \cdots \wedge h_n \otimes (h_i x - x h_i).
\end{equation}

That is, the complex

\begin{equation}
\cdots \wedge^2 \mathfrak{h} \otimes U\mathfrak{h} \to \mathfrak{h} \otimes U\mathfrak{h} \to U\mathfrak{h} \to \mathbb{K} \to 0
\end{equation}
gives us the desired free resolution of $\mathbb{K}$. For an $\mathfrak{h}$ module $M$, the Lie algebra homology is defined as

\begin{equation}
H_*(\mathfrak{h}, M) := \mathbb{K} \otimes_{U\mathfrak{h}}^L M \cong (C_*(\mathfrak{h}) \otimes_{\mathbb{K}} U\mathfrak{h}) \otimes_{U\mathfrak{h}} M = C_*(\mathfrak{h}) \otimes_{\mathbb{K}} M.
\end{equation}
This construction can easily be extended to a dgla \( \mathfrak{h} \). Note that if \( \mathfrak{h} \cong \mathfrak{g} \), then \( C_*(\mathfrak{h}) \cong C_*(\mathfrak{g}) \).

Next, consider the local dgla on an open \( U \subset \mathbb{R} \), given by
\[
\mathcal{L}(U) = \Omega_c(U) \otimes \mathfrak{g}
\]
where the bracket is the Lie bracket on \( \mathfrak{g} \), and the differential is the de Rham differential on forms. The factorization envelope is the prefactorization algebra defined by
\[
\mathbb{U}_\mathfrak{g} : U \mapsto H_*(C_*(\mathcal{L}(U))).
\]
Note that any factorization algebra of this type is locally constant, since the Poincare lemma says that \( \Omega_c(U) \rightarrow \Omega_c(V) \) is a quasi-isomorphism for an inclusion of contractible open sets \( U \subset V \), that is
\[
\Omega_c(U) \simeq \mathbb{C}[-1]
\]
Thus, as vector spaces
\[
C_*(\Omega_c(U) \otimes \mathfrak{g}) \cong C_*(\mathfrak{g}[-1]) = \text{Sym}(\mathfrak{g})[0]
\]
We claim that \( \mathbb{U}_\mathfrak{g}(\mathbb{R}) = U_\mathfrak{g} \). To show this, we construct map of Lie algebras, \( \Phi : \mathfrak{g} \rightarrow C_*(\Omega_c^*(\mathbb{R}) \otimes \mathfrak{g}) \), sending \( X \mapsto \epsilon_a \otimes X \), where \( \epsilon_a \in \Omega^1_c(\mathbb{R}) \) satisfies \( \int \epsilon = 1 \) and is a smooth bump form centered at \( a \), and has support in some small interval \( (a - \delta, a + \delta) \). Then, we can check that \( \epsilon_1 - \epsilon_{-1} = dh \) where \( h \in \Omega^0_c(\mathbb{R}) \), and \( h = -1 \) near 0. then
\[
(d_{DR} + d_{CE})(\epsilon_0 \otimes X)(h \otimes Y) = (\epsilon_0 \otimes X)(d_{DR}h \otimes Y) + \epsilon_0 h \otimes [X,Y]
\]
\[
= (\epsilon_0 \otimes X)((\epsilon_1 - \epsilon_{-1}) \otimes Y) - \epsilon_0 \otimes [X,Y]
\]
\[
= \Phi(X)\Phi(Y) - \Phi(Y)\Phi(X) - \Phi([X,Y])
\]
So \( \mathbb{U}_\mathfrak{g}(\mathbb{R}) = U_\mathfrak{g} \). In this way, we find that the enveloping algebra \( U_\mathfrak{g} \) is naturally constructed from the local dgla of \( \mathfrak{g} \)-valued forms on intervals. In general, for any dgla \( \mathfrak{g} \), the map
\[
\mathbb{U}_\mathfrak{g} : U \mapsto C_*(\Omega^*_c(U) \otimes \mathfrak{g})
\]
defines a locally constant factorization algebra on any manifold \( M \), called the factorization envelope of \( \mathfrak{g} \).

2. FACTORIZATION ALGEBRAS

The data of a prefactorization algebra defines a precosheaf \( U \mapsto \mathcal{F}(U) \). For this to define a factorization algebra, we need this precosheaf to be a cosheaf with respect to a special topology.

Recall, \( \mathcal{F} \) is an ordinary cosheaf on \( X \) if for any open set \( U \subset X \) and cover \( \{U_i\}_{i \in I} \) of \( U \) that the following sequence
\[
\bigoplus_{i,j} \mathcal{F}(U_i \cap U_j) \rightarrow \bigoplus_k \mathcal{F}(U_k) \rightarrow \mathcal{F}(U) \rightarrow 0
\]
is exact.
A prefactorization algebra $F$ is a factorization algebra if $F$ satisfies the cosheaf property above for a more restrictive class of covers on $X$.

**Definition 2.1.** An open cover $\{U_i\}_{i \in I}$ of $U$ is a *Weiss* cover, if for every finite collection of points $\{x_j\} \subset U$, there exists an element $U_k$ of the cover such that $\{x_j\} \subset U_k$.

**Remark 2.2.** (1) Suppose $\{U_i\}_{i \in I}$ is a Weiss cover of $U$. Then, the collection $\{U_i \times U \times \cdots \times U\}_{i \in I}$ is an open cover (in the ordinary sense) of the product space $U \times \cdots \times U$. This implies that a Weiss cover on $X$ induces a topology on the Ran space, the collection of all finite subsets of $X$, Ran($X$). This leads to another characterization of a factorization algebra: a factorization algebra on $X$ is equivalent to a cosheaf on the Ran space.

(2) Weiss covers, in general, are huge. For instance, $\{\mathbb{R} \setminus q \mid q \in \mathbb{Q}\}$ form a Weiss cover of $\mathbb{R}$.

**Definition 2.3.** A *factorization algebra* on $X$ is a prefactorization algebra $F$ on $X$ that satisfies:

- for any open set $U \subset X$ and Weiss cover $\{U_i\}$ of $U$, the sequence (17) is exact;
- for any $U \sqcup V \subset X$ the natural map $F(U) \otimes F(V) \to F(U \sqcup V)$ is an isomorphism.

All our examples so far will be either of the form $U \to \text{Sym}(\mathcal{E}_c(U))$, where $\mathcal{E}_c$ is the space of compactly supported section of a graded vector bundle, or deformations thereof. These examples are automatically multiplicative factorization algebras, due to the compactly supported nature, and the multiplicative behavior of the ring of functions, e.g.

$$\text{Sym}(\Omega_c(U \sqcup V)) = \text{Sym}(\Omega_c(U) \oplus \Omega_c(V)) = \text{Sym}(\Omega_c(U)) \otimes \text{Sym}(\Omega_c(V))$$

We note that to build locally constant factorization algebras on $S^1$, we assign the factorization algebra $A^{lact}$ to $S^1 - \{0\} \cong \mathbb{R}$, and then are attach an $A - A$ bimodule $M$ to an interval containing 0. The defect at the point captures the monodromy of the factorization algebra around the circle.

**2.0.1. Gluing up to homotopy.** For factorization algebras valued in chain complexes $\text{Ch}$, it is more natural to use the homotopy cosheaf condition. Let $F$ be a precosheaf valued in $\text{Ch}$, and $U = \{U_i\}$ an open cover for $U$. Define the Čech complex $\check{C}(U, F)$ as follows. As a graded vector space it is

$$\bigoplus_{k=1}^{\infty} \bigoplus_{j_1, \ldots, j_k \in I, j_i \text{ distinct}} F(U_{j_1} \cap \cdots \cap U_{j_k})[k - 1]$$
The differential is given by the totalization of the internal differential $d_F$ on $F$ and the Čech differential $d_{Čech}$.

**Definition 2.4.** A factorization algebra on $X$ valued in chain complexes is a prefactorization algebra $F$ on $X$ valued in chain complexes satisfying:

1. for every open $U \subset X$ and Weiss cover $\mathcal{U}$ of $U$ the natural map
   $$\check{C}(\mathcal{U}, F) \to F(U) = \int_U F$$
   is a quasi-isomorphism;
2. for any $U \sqcup V \subset X$ the natural map
   $$F(U) \otimes F(V) \to F(U \sqcup V)$$
   is a quasi-isomorphism.

**Remark 2.5.** The Čech complex $\check{C}(\mathcal{U}, F)$ is nothing but a model for the homotopy colimit
$$\hocolim_{U \in \mathcal{U}} F(U).$$

**2.1. Locally constant factorization algebras.** We have formulated the notion of a factorization algebra as an algebra over a colored operad of disjoint disks $\text{Disj}_M$ satisfying a certain gluing condition.

A simplification of the above operad appears when we consider locally constant factorization algebras.

Let $E_n$ be the operad (pseudo-tensor category with one object) of little $n$-disks. The $k$-ary morphisms are given by the the space of framed embeddings of disjoint disks in $\mathbb{R}^n$
$$\sqcup_{i \leq k} D^n \hookrightarrow D^n.$$ Composition is defined in the obvious way. $E_n$ objects in vector spaces or chain complexes are referred to as $E_n$-algebras. The generalization of the earlier result equating locally constant factorization algebras on $\mathbb{R}$ and associative algebras, is due to Lurie:

**Theorem 1 (Lurie [Lur]).** There is an equivalence of $(\infty, 1)$-categories
$$\text{Fact}_{l.c.}(\mathbb{R}^n) \simeq E_n-\text{Alg}$$

Note that commutative algebras give factorization algebras on every $\mathbb{R}^n$, since they are naturally $E_\infty$-algebras.

**3. Factorization Homology**

We can push-forward a factorization algebra $F$ over a continuous map $p : X \to Y$,

$$(p_*F)(U) = F(p^{-1}(U)), \quad U \subset Y.$$
We define the factorization homology, $\int_X F$, for any factorization algebra $F$, as push-forward to a point $f : X \to \text{pt}$, or equivalently taking global sections

\begin{equation}
\int_X F = F(X).
\end{equation}

We explain just some simple examples of how to calculate $\int_X F$.

### 3.1. Hochschild homology.
Consider the factorization algebra $F_{(M,A,N)}$ on $[0,1]$, which assigns the associative algebra $A$ to opens of the form $(a,b)$, assigns the right $A$-module $M$ to opens of the form $[0,a)$, and assigns the left $A$ module $N$ to opens $(a,1]$. We then have the following technical lemma, due to Gwillam ([Gwi12])

\begin{equation}
F_{(M,A,N)}([0,1]) = M \otimes_A^L N
\end{equation}

This is proven using the cosheaf Cech description of factorization homology. However, we can gain some insight into this result by considering the Weiss cover given by the open sets $U_x = [0,1]/\{x\}$. Its easy to see that

\begin{equation}
F_{(M,A,N)}(U_{x_0}) = M \otimes N
\end{equation}

and

\begin{equation}
F_{(M,A,N)}(U_{x_0} \cap U_{x_1}) = M \otimes A \otimes N
\end{equation}

All the possible maps give the contributions to the differentials of the bar complex. E.g. the first structure map in the sequence

\begin{equation}
F_{(M,A,N)}(U_{x_0} \cap U_{x_1}) \to F_{(M,A,N)}(U_{x_0}) \oplus F_{(M,A,N)}(U_{x_1}) \to F_{(M,A,N)}([0,1])
\end{equation}

is

\begin{equation}
m \otimes a \otimes n \mapsto (ma \otimes n, -m \otimes an)
\end{equation}

All the maps in the construction of the ‘bar’ complex construction of $M \otimes_A^L N$ appear in this way.

Now, consider the locally constant factorization algebra $A^{\text{fact}}$ on $S^1$, built from an associative algebra $A$. We push-forward this factorization algebra with the map $p = \sin : S^1 \to [-1,1]$. Then $p_* A^{\text{fact}}$ is characterized by $p_* A^{\text{fact}}((a,b)) = A \otimes A^{op}$, and we can see that

\begin{equation}
p_* A^{\text{fact}} = F_{(A, A \otimes A^{op}, A)}
\end{equation}
Hence, we have

\begin{align*}
A^{\text{fact}}(S^1) &= \pi_* A^{\text{fact}}([-1, 1]) \\
&= \mathcal{F}_{(A_A \otimes A_{\text{op}}, A)}([-1, 1]) \\
&= A \otimes_{A_{\text{op}}} A \\
&\cong HH^*_*(A)
\end{align*}

Instead, we could consider the factorization algebra on $S^1$ where we have glued the ends of $\mathbb{R} = S^1 - \{0\}$ together using the twisted bimodule $A_\sigma$ at $\{0\}$. In this case we would find

\begin{equation}
A^{\text{fact}}_\sigma(S^1) = A \otimes_{A_{\text{op}}} A_\sigma \cong HH^*_*(A, A_\sigma)
\end{equation}

Lastly, we use the de Rham description of factorization homology to compute the Hochshild homology of the enveloping algebra. We know that

\begin{equation}
U^*_g(S^1) = HH^*_*(U^g).
\end{equation}

This follows from the Cech computations above. However, we can also perform this calculation using the de Rham model. We begin by noting that $S^1$ is formal, i.e. $\Omega^*(S^1) \cong H^*(S^1) = \mathbb{C}[0] \oplus \mathbb{C}[-1]$ as dg assoc algebras, since there are no higher operations in cohomology (Massey products). Thus, $C_*(\Omega^*(S^1) \otimes g) \cong C_*(g \otimes \mathbb{C}[-1]) = C_*(g) \otimes \text{Sym}(g)$ as chain complexes. As before, we can easily check that the $g$-module action on $\text{Sym}(g)$ agrees with that of $U^g$, and we find that the Hochshild homology of $U^g$ computes the Lie algebra homology of the $U^g$ as a $g$-module,

\begin{equation}
HH^*_*(U^g) = H_*(C_*(g) \otimes U^g) = H_*(g, U^g).
\end{equation}

**References**

