Topics in Topology: Knots and Three-Manifolds

TTh 5:50 – 7:20 pm, Ryder Hall 394

www.neu.edu/beasley/MATH7375

Course Instructor

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Office hours: Tuesday 4:00 - 5:30 pm, Thursday 10:30 am - 12:00 pm

Other times by appointment.

Course Description

This course serves as an introduction to the study of knots and three-manifolds. Questions about the topology of manifolds in dimension three seem to occupy a 'sweet spot' between triviality and intractability. In the first half of the course, we discuss a variety of ways to distinguish knots and construct three-manifolds. In the second half of the course, we refine these notions to study contact topology (the odd cousin of symplectic topology) in dimension three.

Prerequisites: Permission of instructor. Like number theory, knot theory is a deep subject which can be developed from only elementary ideas. I will assume a basic knowledge of algebraic topology (fundamental group, co/homology, Poincaré duality) as well as an acquaintance with smooth manifolds and differential forms, but the lectures will be otherwise self-contained. Interested first-year students are encouraged to contact the instructor directly for any questions about prerequisites.

Textbook

Part I of the course will be based on material in the following references.

- W. Lickorish, *An Introduction to Knot Theory*, Springer, 1997.
- V. Prasolov and A. Sossinsky, *Knots, Links, Braids, and 3-Manifolds: An Introduction to the New Invariants in Low-Dimensional Topology*, Trans. of Math. Monographs **154**, AMS Publishing, 1997.
- D. Rolfsen, *Knots and Links*, AMS Publishing, 1976.

Part II of the course will follow selected chapters in the textbook by Geiges.

- H. Geiges, *An Introduction to Contact Topology*, Cambridge University Press, 2008.

These references will be available on 3-hour reserve in Snell Library. I will also post scanned pdf lecture notes to the course website each week.

Important Topics Not Included

Topology in dimension three is a vast subject, impossible to survey in a single semester. Though I hope to mention fundamental structural results, the course will omit any detailed discussion of Thurston geometrization and special techniques to analyze hyperbolic three-manifolds. The course will also omit any discussion of foliations or homological invariants of knots and three-manifolds.

Grade

The course grade will be based upon class participation. As a means to practice mathematical exposition, each student will also be asked to produce complete and polished TeX versions of selected lectures, including problems which may be assigned during those lectures.

Other Course Policies

General course policies apply as described in the document below.

www.neu.edu/cos/wp-content/uploads/2014/11/Northeastern-COS-Policies-Template.pdf

If you have a concern about the course, I am always happy to meet with you about it. If the issue is still not resolved, you can discuss it with the Graduate Coordinator, Prof. Maxim Braverman, `m.braverman@neu.edu`, 467 Lake, x8769.

Course Outline

Week of	Topics Covered
1/11	Introduction to knot theory. Plane projections and Reidemeister moves.
1/18	Knot group and Wirtinger presentation.
1/25	Three-dimensional topology. Theorems of Papakyriakopoulos.
2/1	Seifert surfaces and knot genus.
2/8	Cyclic coverings and Alexander invariants.
2/15	Jones and HOMFLY polynomials. Vassiliev invariants.
2/22	Three-manifolds and surgery on links.
2/29	Other presentations for three-manifolds.
3/7	Spring Break.
3/14	Introduction to contact structures and Reeb fields.
3/21	Contact manifolds: generalities.
3/28	Legendrian and transverse knots.
4/4	Contact structures on three-manifolds.
4/11	Surfaces in contact three-manifolds.
4/14	Open books and Giroux correspondence.