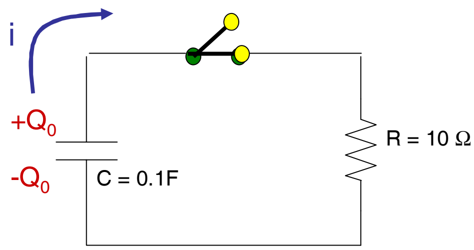
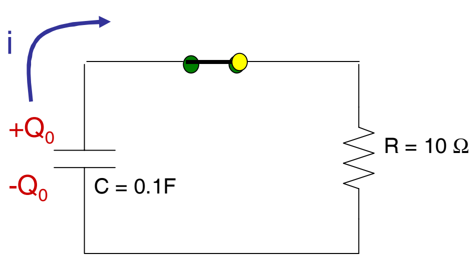
RC circuits

Initially one has $+Q_0$ and $-Q_0$ on the Capacitor plates.
Thus, the initial Voltage on the Capacitor $V_0 = Q_0/C$.
What do you think happens when the switch is closed?



Discharging a capacitor

Close the switch at t=0, then current i starts to flow.

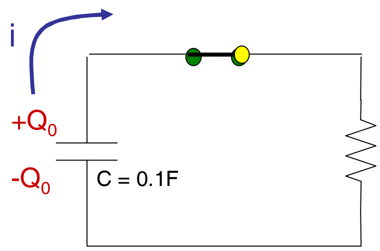


At t=0, $i_0 = \dfrac{V_0}{R}$

Later $i(t) = -\dfrac{dQ}{dt}$

* Negative sign since Q is decreasing.

Discharging a capactiror



Voltage across C = Voltage across R

$$V_c = V_R$$

$$\frac{Q}{C} = iR = -\frac{dQ}{dt}R$$

$$\frac{dQ}{dt} = -\frac{1}{RC}Q$$

We now need to solve this differential equation.

Discharging a capacitor

Solving the differential equation:

$$\frac{dQ}{dt} = -\frac{1}{RC}Q \quad \Rightarrow \quad Q(t) = Q_0 e^{-t/RC}$$

Check solution by taking the derivative...

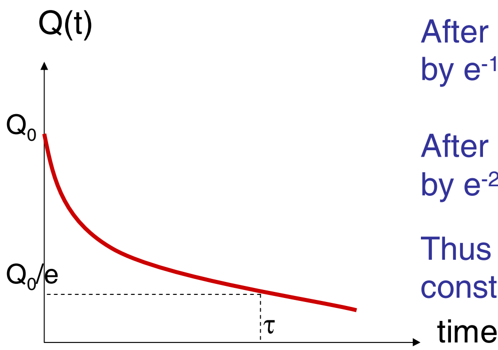
$$\frac{dQ}{dt} = Q_0\left(-\frac{1}{RC}\right)e^{-t/RC} = -\frac{1}{RC}Q$$

Also Q(t) = Q_0 at t=0. $Q(t=0) = Q_0 e^{-0/RC} = Q_0$

Discharging a capacitor

Exponential Decay

$$Q(t) = Q_0 e^{-t/RC}$$



After a time τ = RC, Q has dropped by $e^{-1} = 1/e$.

After a time t = 2RC, Q has dropped by $e^{-2} = 1/e^2$

Thus τ=RC is often called the time constant and has units [seconds].

Discharging a capacitor

$$\frac{dQ}{dt} = Q_0\left(-\frac{1}{RC}\right)e^{-t/RC} = -\frac{1}{RC}Q$$

$$|\,i(t)\,| = \left|\frac{dQ}{dt}\right| = \left(\frac{Q_0}{RC}\right)e^{-t/RC} = i_0 e^{-t/RC}$$

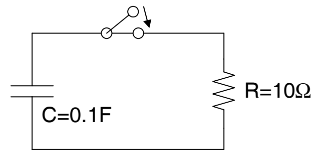
Thus, the current also falls with the same exponential function.

Clicker Question

A capacitor with capacitance 0.1F in an RC circuit is initially charged up to an initial voltage of V_o = 10V and is then discharged through an R=10Ω resistor as shown. The switch is closed at time t=0. Immediately after the switch is closed, the initial current is $I_o = V_o / R = 10V/10\Omega$.

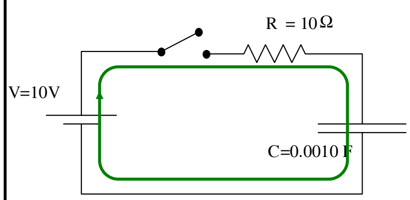
What is the current I through the resistor at time t=2.0 s?

A) 1A
B) 0.5A
C) 1/e A = 0.37A
D) $1/e^2$ A = 0.14A
E) None of these.



Charging a capacitor

More complex RC circuit: Charging C with a battery.



Before switch closed i=0, and charge on capacitor Q=0.

Close switch at t=0.

Try Voltage loop rule.

$$+V_b + V_R + V_C = 0$$

$$+V_b - iR - Q/C = 0$$

Charging a capacitor

$$+V_b - iR - Q/C = 0$$

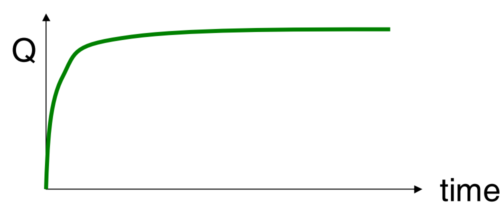
$$+V_b - \frac{dQ}{dt}R - Q/C = 0$$

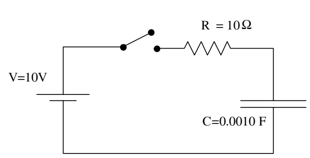
$$\frac{dQ}{dt} = +\frac{V_b}{R} - \frac{Q}{RC} \quad \Rightarrow \quad Q(t) = CV_b\left(1 - e^{-t/RC}\right)$$

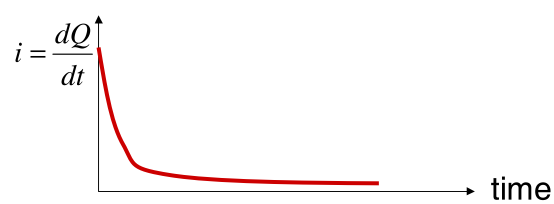
Charging a capacitor

$$Q(t) = CV_b\left(1 - e^{-t/RC}\right)$$

$$i(t) = \frac{dQ}{dt}(t) = \frac{V_b}{R}e^{-t/RC}$$







- Although no charge actually passes between the capacitor plates, it acts just like a current is flowing through it.
- Uncharged capacitors act like a "short": V_C=Q/C=0
- Fully charged capacitors act like an "open circuit". Must have i_C = 0 eventually, otherwise Q → infinity.

Clicker Question

An RC circuit is shown below. Initially the switch is open and the capacitor has no charge. At time t=0, the switch is closed. What is the voltage across the capacitor ***immediately*** after the switch is closed (time = 0)?

A) Zero B) 10 V
C) 5V D) None of these.