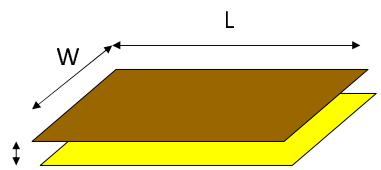
Capacitor

Any two pieces of metal (conductor) brought near each other.

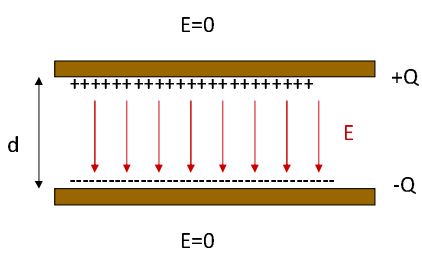
Parallel plate capacitor



Area A = L x W

Charged capacitor

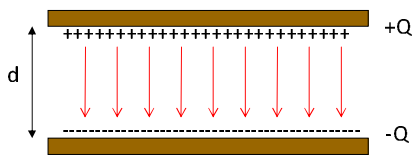
"Charged Capacitor" has no <u>net</u> charge.



$$E = |\vec{E}| = \frac{\sigma}{\varepsilon_0} = \frac{1}{\varepsilon_0}\left(\frac{Q}{A}\right)$$

Electric Field (treating plates as infinite) and having area A.

Capacitance



$$E = \frac{\sigma}{\varepsilon_0} = \frac{1}{\varepsilon_0}\left(\frac{Q}{A}\right)$$

$$V = \Delta V = \int \vec{E} \cdot d\vec{r} = Ed$$

$$\rightarrow \boxed{V = \frac{Q}{(\varepsilon_0 A/d)}}$$

We define **"capacitance"** $\boxed{C \equiv \frac{Q}{V}}$

For parallel plate capacitor $\boxed{C \equiv \frac{\varepsilon_0 A}{d}}$

Capacitance

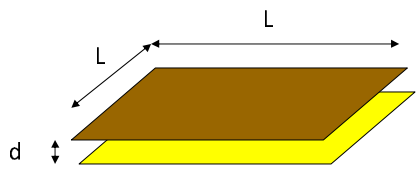
$\boxed{C \equiv \frac{Q}{V}}$

Can think of capacitor as a device for storing charge (and also energy).

Units [Capacitance] = [Coulomb/Volt] = [Farad]

CPS Question

A parallel-plate capacitor has square plates of edge length L, separated by a distance d.
If we doubled the dimension L and halve the dimension d, by what factor have we changed the capacitance?



$$C = \frac{\varepsilon_o A}{d} = \frac{\varepsilon_o L^2}{d} \rightarrow \frac{\varepsilon_o (2L)^2}{d/2} = 8\frac{\varepsilon_o L^2}{d}$$

A) 1.0
B) 2.0
C) 4.0
D) 8.0
E) 16.0

CPS question

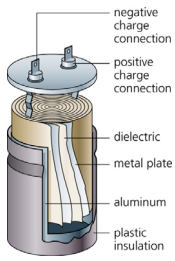
How big is a one Farad Capacitor?

$$C \equiv \frac{\varepsilon_0 A}{d}$$

Assume you have two parallel plates that are separated by 1 mm. If you estimate $\varepsilon_0 \sim 10^{-11}$, what is the area of the plates to have a 1.0 Farad capacitance?

A) 100,000,000 square meters
B) 1,000 square meters
C) 1 square meters
D) 0.1 square meters
E) 0.001 square meters

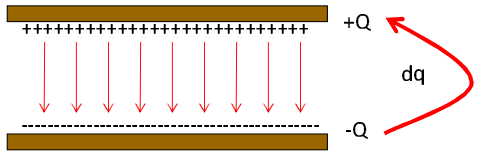
Commercial capacitors



Micro-farad capacitors in small packages are made by making d very small ~ atomic dimensions.

Energy Storage

Capacitor stores both charge and <u>energy</u>.
Suppose the capacitor is not fully charged. How much work is required to move a little bit $+dq$ across?



$$Work = dW = dU = Vdq$$

$$dU = \left(\frac{q}{C}\right)dq$$

$$U = \int dU = \int_0^Q \left(\frac{q}{C}\right)dq = \frac{Q^2}{2C}$$

Each next piece +dq gets harder and harder.

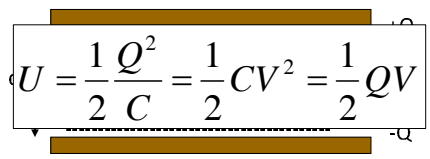
$$\boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV}$$

Potential Energy stored in the capacitor.

CPS Question

A parallel plate capacitor is charge up (+Q on one plate and –Q on the other). The plates are isolated so the charge Q cannot change. The plates are then pulled apart so that the plate separation d increases. The total electrostatic energy stored in the capacitor?

A) Increases
B) Decreases
C) Remains constant



$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Electric Field energy

Where is this energy U?

Answer = the electric field has energy.

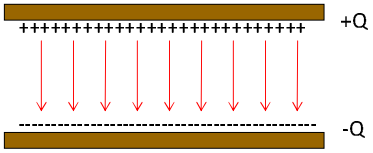
Energy density of E-field = $\frac{energy}{volume} = \frac{U}{Vol.} = u = \frac{\frac{1}{2}CV^2}{Vol.} = \frac{\frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2}{A \cdot d}$

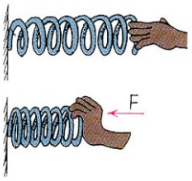
$$= \frac{1}{2}\varepsilon_0 E^2$$

Turns out work to charge capacitor is equal to energy stored.

This is valid for any electric field configuration in vacuum: "empty" space contains energy!!!

Spring analogy





$$U = \frac{1}{2}\left(\frac{1}{C}\right)Q^2$$

Electric field energy

$$U = \frac{1}{2}kx^2$$

Elastic energy

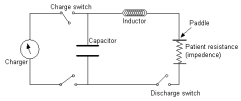
Applications

What is this device?

1. **Energy used for cardioversion & defibrillation**
   a) Electrical output of defibrillators is expressed in terms of energy
   b) Joules (J) or watt-seconds (Ws) describe the power (watts) and the length of time it is applied (sec)
   c) Thus Energy (Joules) = power (watts) × duration (seconds)
   d) note: Watt = current (amps) × voltage (volts)
      watt = 1 J sec^{-1}
   e) Defibrillators are set according to the amount of energy stored; this depends on both the charge & the potential
   f) Potential [V] = $\frac{\text{Power[W]}}{\text{Current [A]}}$
   g) Power (W) = Energy (J) per second
   h) Current(A) = Charge(C) per second
      i) $V = \frac{Js^{-1}}{Cs^{-1}} = \frac{J}{C}$
      ii) J = CV
      iii) Stored energy (J) = 1/2 X stored energy (C) x Potential (V)
      iv) EG: potential of 5000 V is applied across two plates of a capacitor, produces a store of electrons of 160mC of charge
      v) Energy = 1/2 x 160 mC x 5000 V = 400 J

Capacitance is the measure of the ability of an object to hold an electrical charge.
SI unit is Coulomb (C)
Coulomb = Amperes (A) x seconds (s)

2. **Operation**
   a) Defibrillation energy is temporarily stored in a capacitor
   b) The large capacitor is charged to the selected energy and then discharged through the paddles applied to the chest
      i) The energy stored in the capacitor is released as a current pulse (eg. 35 A for 3 ms) resulting in a synchronous contraction of the heart after which a refractory period and normal beats may follow
   c) Inductor
      i) Included in circuit to ensure that the electric pulse has an optimal shape & duration
      ii) During discharge, the inductor absorbs some of the energy so that not all is discharged to the patient
         a) Defibrillators are calibrated in terms of delivered energy not stored energy

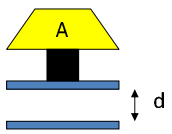


Applications (cont)

Capacitors are good for storing large amounts of energy, which can then be accessed quickly (e.g. camera flash).

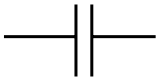
Capacitors are also ideal <u>transducers</u>.

Devices that covert physical quantities into electrical signals (e.g. computer keyboard, elevator buttons).



$$C \equiv \frac{\varepsilon_0 A}{d}$$

Circuits with capacitors



Circuit symbol for a capacitor

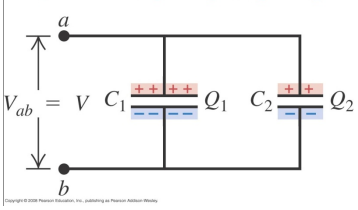
More than one capacitor?...

Capacitors in parallel

(a) Two capacitors in parallel

Capacitors in parallel:

- The capacitors have the same potential V.
- The charge on each capacitor depends on its capacitance: $Q_1 = C_1V$, $Q_2 = C_2V$.



$$V_1 = V_2 = V_{ab} = V$$

$$\rightarrow Q_1 = C_1V; \quad Q_2 = C_2V$$

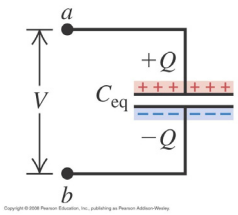
$$Q_{total} = Q_1 + Q_2 = (C_1 + C_2)V$$

$$\rightarrow \frac{Q}{V} = C_1 + C_2$$

$$\rightarrow \boxed{C_{eq} = C_1 + C_2}$$

Capacitors in parallel (cont)

(b) The equivalent single capacitor



Charge is the sum of the individual charges:
$Q = Q_1 + Q_2$

Equivalent capacitance:
$C_{eq} = C_1 + C_2$

Think of two capacitors in parallel as adding the area of the two capacitors

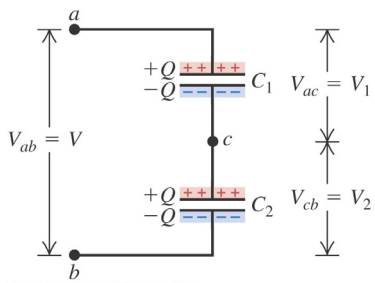
Many capacitors in parallel:

$$C_{eq} = C_1 + C_2 + C_3 + \ldots$$

Capacitors in series

(a) Two capacitors in series

Capacitors in series:
- The capacitors have the same charge Q.
- Their potential differences add: $V_{ac} + V_{cb} = V_{ab}$.



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$$Q_1 = Q_2$$

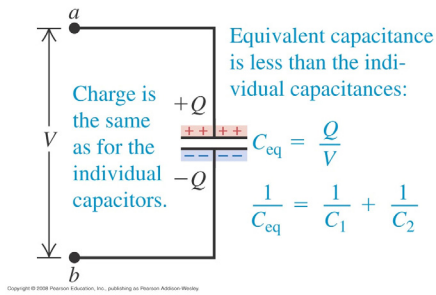
$$\rightarrow V_{ac} = V_1 = \frac{Q}{C_1}; \quad V_{cb} = V_2 = \frac{Q}{C_2}$$

$$V_{ab} = V = V_1 + V_2 = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\rightarrow \frac{V}{Q} = \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Capacitors in series (cont)

(b) The equivalent single capacitor



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$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

or

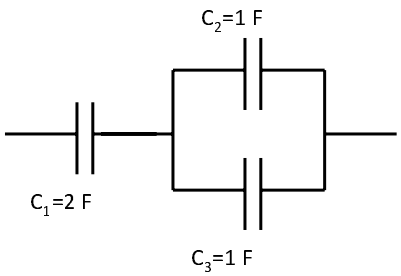
$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

Many capacitors in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

CPS question

A bank of three capacitors (C_1, C_2, C_3) is connected as follows.

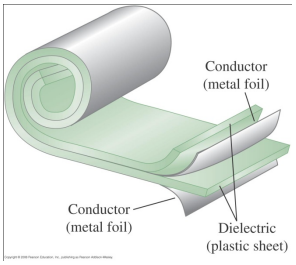


What is the effective total capacitance of the bank?

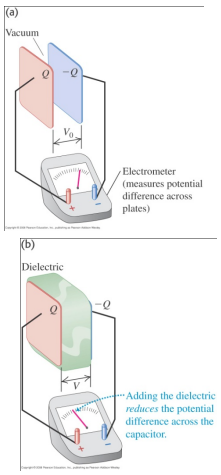
A) C = 0 Farad
B) C = 1 Farad
C) C = 1.5 Farad
D) C = 2.0 Farad
E) C = 0.5 Farad

Dielectrics

What if the gap between the capacitor plates is not empty?



Dielectrics (cont)



Without dielectric:

$$C_{without} = Q / V_0$$

With dielectric:

$$C = Q / V$$

$$\text{but} \quad V < V_0$$

$$\rightarrow C > C_{without}$$

Increases the capacitance!!!

Dielectric constant

Dielectric constant:

$$K = \frac{C}{C_0} = \frac{V_0}{V}$$

$$V = \frac{V_0}{K}$$

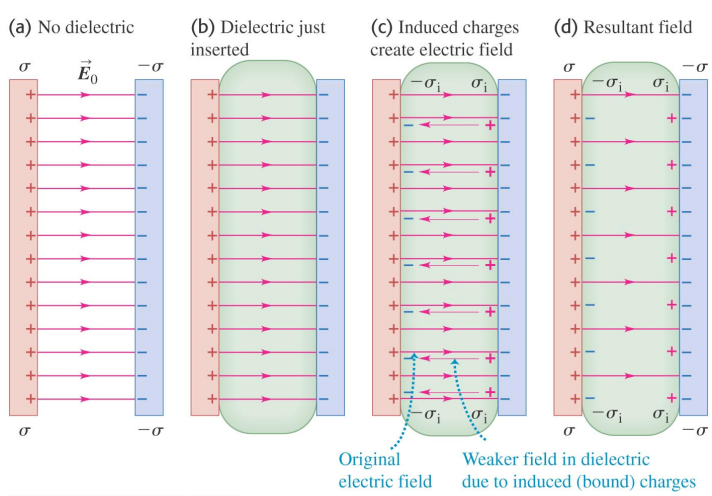
With the dielectric present, the potential difference for a given charge Q is reduced by a factor K

Dielectric constants for some materials

Table 24.1 Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

Induced charge and polarization



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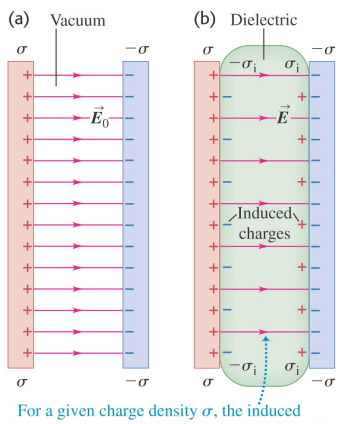
Polarization



For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

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We assume that the induced surface charge density is directly proportional to the electric field magnitude in the material

Without dielectric: $E_0 = \dfrac{\sigma}{\varepsilon_0}$

With dielectric: $E = \dfrac{E_0}{K} = \dfrac{\sigma - \sigma_{induced}}{\varepsilon_0}$

From these we obtain: $\boxed{\sigma_{induced} = \sigma\left(1 - \dfrac{1}{K}\right)}$

Permittivity

We have found: $E = \dfrac{E_0}{K} = \dfrac{\sigma}{K\varepsilon_0}$

Defining: $\varepsilon = K\varepsilon_0$ permittivity

We obtain: $E = \dfrac{\sigma}{\varepsilon}$

and $C = KC_0 = K\varepsilon_0 \dfrac{A}{d} = \varepsilon \dfrac{A}{d}$ Capacitance

$u = \dfrac{1}{2} K\varepsilon_0 E^2 = \dfrac{1}{2}\varepsilon E^2$ Energy density

Easy rule of thumb: replace ε by ε_0 in all the equations