Juggling and Bouncing Balls: Parallels and Differences in Dynamic Concepts and Tools

DAGMAR STERNAD

Department of Kinesiology, Pennsylvania State University, USA

Two lines of research on the related skills of rhythmic bouncing a ball and juggling three balls were reviewed with the goal to reveal commonalities in the strategy typifying the dynamic systems approach to movement coordination. For both lines of research concepts from nonlinear dynamics and their experimental results were presented in parallel. While there were evident differences in the physical principles and key variables, the dual presentation demonstrated the similarity in the modeling strategy and the methodology. Criteria for dynamically stable solutions defined the boundaries that the actor’s movements had to satisfy in order to perform the rhythmic task. Essential in both approaches was that one key variable provided the reference frame for evaluating skilled performance and the process of learning. The role of perceptual information was evaluated by the decrement in stability of performance that selected perceptual manipulations induced. Individual differences between subjects in ball bouncing were shown to be the consequence of their choice of the key variable, which further constrained the kinematic properties across different movement realizations. Individual differences between expert jugglers were interpreted as their “deliberate” choice of a solution that was not tightly constrained by maximum stability but rather one that afforded them more flexibility. This contrasting review aimed to show the spectrum of tools that a dynamic approach has to offer. It further showed that an analysis from a nonlinear dynamic perspective can establish a basis from which a set of important issues in motor control can be addressed, in a qualitative and physically principled manner.

KEY WORDS: Ball perception, Dynamic system perspective, Motor control, Perceptual information.

Bouncing and Juggling Balls: Two Different Tasks?

Imagine holding a tennis racket in your hand and hitting a ball repeatedly up in the air, as shown in Figure 1, i.e., you “juggle” the ball rhythmically, so

Address for correspondence: Dagmar Sternad, Department of Kinesiology, Pennsylvania State University, 266 Recreation Building, University Park, PA 16802

462
that the ball bounces with an approximately consistent height. Apparently, this is a rather simple skill because no serious practice is required, and you have no problems walking and talking while you are bouncing the ball. And yet, a closer look (or the inquisitive eye of a movement scientist) immediately reveals that this single-handed juggling task holds all the archetypal challenges of perceptual-motor coordination. To propel the ball to a specific amplitude requires a well-defined velocity of the racket at impact, and a certain angle of the racket surface is needed to keep the ball’s trajectory vertical and, thereby, confined in space. Swinging the racket rhythmically up and down requires moving the whole arm, which involves the coordination of the kinematic chain of shoulder, elbow and wrist joints. Also the fundamental issues in perception are obvious: What kind of information is necessary to hit the ball at the right time and the right place in order to attain the rhythmic ball bouncing? Is it necessary to continuously follow the ball’s trajectory with your eyes in order to «prepare for» the right velocity at impact? Also, every amateur tennis player knows that feeling the «sweet spot», i.e., haptic information about the impact, provides crucial information about the success or failure of a tennis stroke. That this task has interesting perceptual and dynamic properties has been demonstrated in previous and ongoing research in our laboratory (Schaal, Sternad, & Atkeson, 1996; Sternad, Katsumata, Duarte, & Schaal, 1999; Sternad, Duarte, Katsumata, & Schaal, submitted).

Fig. 1. - (A) The task of bouncing a ball with a racket. (B) The mechanical model consisting of a planar surface and a ball. The formulation of the model equation includes arbitrary periodic motions of the surface, inelastic impact of ball and surface, and ballistic flight of the ball between impacts.

Let's change the game. Instead of hitting the ball with a racket, imagine you toss and catch one ball repeatedly up in the air. What was in the bouncing task a short and critical ball contact, now becomes a controlled event involving the manipulation of the ball with all fingers of the hand in order to achieve
a smooth catch and a controlled throw. Identical to the bouncing task, though, the velocity and the angle of ball release uniquely defines the ball’s ballistic flight trajectory. Since this task is still fairly easy, let’s make it more difficult and use more than one ball and let’s use both hands. To be specific, we want to use three balls which are to be thrown in a figure-8 pattern (Figure 2). This modification leads to so-called cascade juggling, a skill which was extensively studied in a series of experiments by PJ. Beek and colleagues (Beek, 1988; Beek & van Santvoord, 1996; Beek & Turvey, 1992). Cascade juggling is a complex cyclic action in which the hands move along elliptic trajectories, one clockwise, the other anti-clockwise, and the balls are released at the inside of the ellipses and caught at the outside. The movements of the hand mirror one another, although with a phase lag close to 180 degrees, and they tightly match the parabolic trajectories of the ball. Compared to the simple bouncing task, juggling not only involves the synchronization of two hands, but the juggler also has to match the angular loop frequency of the hand movements with the frequency of the motion of the balls. There are also spatial constraints: As the number of balls is larger than one, the toss has to propel the balls so that they do not collide in midair. This appears to require an intense monitoring of the balls’ trajectories to avoid such collisions. Yet, professional jugglers can juggle three balls blindly, which means that they have to mainly rely on haptic information. Are these two sources of information interchangible? How do they aid the less experienced juggler to keep the balls up in the air?

Fig. 2. - Sketch of juggling three balls in a figure-8 or cascade pattern. (The sketch is drawn after Beek, 1988.)

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Clearly, the two tasks have differences: Bouncing a ball involves a single manipulator and a single ball, whilst cascade juggling requires two manipulators that juggle three or more objects in the air. In ball bouncing the racket-ball contact is almost instantaneous, whereas in juggling the catch allows more controlled preparation of the next ball's trajectory. On the other hand, the two skills both involve the cyclic manipulation of an object. In this sense, one may argue that the two skills are related but they are of a different level of complexity. One can conceive the almost instantaneous ball-racket impact as the limit case of a catch and toss. Similarly, «one-ball juggling» is the easiest version which can be scaled up to juggling several balls. Juggling with one hand is the beginner's routine, compared to performance that involves two hands or even more effectors as when several people with several hands are involved.

Considering these variations, one can argue that these two skills constitute the extremes of one or even several continua: Ball skills can be different in the kinds of ball contact, or in the kind of body parts that make contact with the ball. For instance, setting a volleyball involves a ball contact which is intermediate between the catching and tossing action with an extended temporal interval and the bouncing action where the contact is close to instantaneous. Due to the volleyball's weight and force at contact the fingers and wrists of both hands are extended, elastic energy is stored and then released in the set. If this action is performed rhythmically, as in the typical warm-up routine, the task can be classified as somewhere between juggling and bouncing. Of course, juggling one or several balls can also be performed with the foot and other body parts. The rhythmic kicking of a soccer ball with one foot, as often seen when boys practice to improve their ball «handling» skill, is close to the above described ball bouncing. Similar, the children's game of repetitively hitting a ball against a wall via a bounce off the floor is another variant of the bouncing task. Similarly, periodic interception of a ball with a racket is also central in games such as table tennis or tennis, where two players «bounce» the ball back and forth.

One definite commonality in the two tasks of juggling and bouncing balls is that they have been both investigated from a dynamic systems perspective. Fundamental to both scientific treatments was that the rhythmic juggling or bouncing act was interpreted as the stable organization of the high-dimensional musculo-skeletal and perceptual system of the juggler. In both lines of inquiry, explanation for the rhythmic performance was sought neither in cognitive constructs and inferential structures, nor in neurophysiological mechanisms. Instead, the goal was to reveal general physical principles that reflect the organization of the human perception-action system within its task specific spatiotemporal constraints. If movement coordination is viewed as a
nonlinear dynamic system, it can be expected that attractive stable regimes exist which become evident in human performance as an intrinsic preference for particular realizations out of many other possible movements. The quantitative treatment concentrated on kinematic regularities measurable at the behavioral level and mathematical models were brought to bear to capture the task and the patterns exhibited by performers.

And yet, despite their common agenda, the level of modeling, the mathematical tools and the physical concepts are remarkably different. The purpose of this review is to present a synopsis of both lines of work, side by side, and thereby reveal the commonalities in strategy in parallel with the differences in dynamic concepts and tools. This is an attempt to bridge across the two tasks and unify what otherwise is often viewed as individualized and separate research agendas. The review’s organization reflects the general strategy of a dynamic systems approach: First, a dynamic analysis of the task is presented where the central concepts of the formal approach are stability and variability. After the fundamental task description, the issue of learning is addressed in which the same formal analysis provides the reference frame for measuring changes in skill level. In close connection with learning resides the question of interindividual differences. On the basis of further formal analyses, answers are suggested that account for differences amongst individuals. Lastly, the perceptual basis for executing the task will be investigated. Each of these steps will be exemplified by reviewing juggling and ball bouncing in tandem, presenting both dynamic concepts and selected experimental data. This strategy will be compared to one other line of work which derived from dynamic pattern approach. In conclusion, speculations are presented whether insights can be transferred to other rhythmic interceptive ball tasks.

The First Step: Capturing the Constraints

Juggling Three Balls

How does the juggler coordinate two hands juggling several balls? A probably unique point of departure for the study of such a complex skill was PJ. Beek’s analysis of the partitioning of hand and ball movements into component times (Beek, 1988, 1989a,b). Following the original formulation by Claude Shannon (who was not only the founder of information theory but also a juggling enthusiast), three such intervals can be distinguished: the average time the ball is in the air between throw and catch, $T_r$ (time flight), the average time a ball is held in the hand between a catch and a throw, $T_l$ (time loaded), and the average time that the ball is performed, e.g., $H=2, (T_l + T_r + T_L + T_T)$. As both times can be calculated $(T_l + T_r)/(T_l)$, that success analyses on these variably timing the hand $H$, always or example, if $T_l$ is pared to $T_u$, throw the balls spatial clothe hold the ball notation, this ratio free of.

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time that the hand is free of a ball, $T_U$ (time unloaded). Given that a pattern is performed with a set number of balls, e.g., $N=3$, and a set number of hands, e.g., $H=2$, (the number can be more than two when more jugglers perform one pattern together), a constraint can be formulated which every juggling performance has to satisfy: If the cycle time of the hand is $T_H$ and $T_H = T_U + T_L$, then for $N$ balls the total loop time $T = N(T_U + T_L)$. The ball's cycle time $T_B$ consists of $T_L$ and $T_F$, $T_B = T_L + T_F$. Because there are two hands, the total loop time is $H(T_L + T_F)$. As both loop times have to be identical for periodic juggling, the two cycle times can be equated: $N(T_U + T_L) = H(T_F + T_L)$, or equivalently: $N/H = (T_F + T_L)/(T_U + T_L)$. This equation precisely formulates the timing constraints that successful juggling has to satisfy. It provides the basis for all the subsequent analyses on what the juggler must control and what he is free to vary. Given these variables, the question is what are the juggler's options and limits in partitioning the hand's cycle into these component times. Given a fixed set of $N$ and $H$, always one of the three time quantities is constrained by the other two. For example, if $T_F$ is fixed, the juggler has the option to lengthen or reduce $T_L$ compared to $T_U$. Indeed, this is of reportedly practical relevance. Jugglers tend to throw the ball to a fixed height, which implies a fixed $T_F$, and thereby set up a «spatial clock». The remaining open issue is then, how long does the juggler hold the ball in hand, $T_L$, relative to the whole hand loop time $T_L + T_U$. Using the notation, this question can be formalized by introducing the notion of a «dwell ratio» expressed in the parameter $k$ which is defined as $T_L/(T_L + T_U)$.

Is there a relative timing that remains invariant regardless of the frequency of execution and of the absolute metrics of the juggling pattern? In short, is there an invariant $k$, and if so, which $k$ do jugglers chose? Clearly, the boundaries for $k$ are 0 and 1. $K$ approaches 0 when the objects are tossed as quickly as possible, often referred to as «hot potato juggling». The average number of balls in the air is $N-Hk$ (for the derivation see Beek, 1988). As the opposite strategy, the toss can be delayed in «delayed juggling», such that $k$ approaches 1. In this case, there are fewer balls in the air (with the limit $N-H$), which is a safer strategy if ball collisions are to be avoided. In preview, this timing ratio $k$ indeed proves to be an invariant which encapsulates the central problem of coordination in juggling.

**Dynamic Underpinnings for Invariant Relative Timing**

The identification of invariants is central in the search for principles in coordination. Specifically invariant relative timing has been discussed as a central feature in a motor program in the motor control literature of the 1970s and
80s (Gentner, 1987; Heuer, 1988; Schmidt, 1980, 1975; Schmidt & Lee, 1998). If the proportional timing of a skill is constant, then it suffices to scale the absolute magnitude by tuning it with a parameter. But, as Beek and others have argued, the identification of invariants is only the start of scientific inquiry and should not be regarded as an end in itself (Beek, 1992; Kelso, 1995). Adopting a dynamic systems view, the next and crucial step is to uncover the physical underpinnings that give rise to such invariant relations, that is to find a principled account as to why one value of $k$ is more preferable than the other.

Central to a dynamic systems analysis is the concept of stability. Motivated by the theory of frequency locking, which states that two coupled oscillations tend to lock into frequencies that form small integer ratios, Beek (1989a,b) posited that the component times $T_L$ and $T_U$ should favor temporal relations where they form small integer ratios, because phase locking is generally known to be more stable there. If $T_L/T_U = 2$, $3/2$, $1$, $1/2$, for $H=2$ and $N=3$, then $k = 1$, $9/10$, $3/4$, $1/2$, etc. On the basis of his initial experimental results which yielded $k$ values in the range between .54 and .83 for a three-ball cascade in expert jugglers, Beek (1989b) argued that $k = 3/4$, or .75, is the primary value that obtains stable frequency locking, using Denjoy’s method of decomposition of frequency modulated waves.

Further theoretical rationale supporting this statement was proposed in the “tiling hypothesis” formulated by Beek and Turvey (1992). A useful strategy to avoid collisions in midair, is to keep the number of balls in the air small, which implies increasing the balls’ dwell time in the hand. $T_L$ should be greater than $T_U$ such that the average number of objects in the air will be less than 2. The time difference between $T_L$ and $T_U$ may therefore be a basic unit of time in the organization of juggling motions. It is consequently hypothesized that the temporal unit $T_L$-$T_U$ is frequency-locked to the handloop time $T_L$+$T_U$, such that the ratio $(T_L+T_U)/(T_L$-$T_U) = W$, where $W$ denotes the so-called winding number. When partitioning the loop times into multiples of $(T_L$-$T_U)$, this “tiling” can be rewritten as $k = 1/2 + 1/2W$ (see Beek & Turvey, 1992). For the smallest winding number $W = 2$, $k$ becomes $3/4$, for $W = 3$, $k = 2/3$, etc.. It suggests that $W = 2$ leads to the most stable performance because it is generally known in nonlinear dynamics that periodic behavior with winding numbers $W$ composed of small integers, e.g., 1:1, 1:2, 1:3, is more stable. Interestingly, this relationship between $W$ and $k$ holds regardless of the number of balls juggled and the number of hands.

Another related line of argument in support of maximal stability at $k = 3/4$ was derived from the theory of nonlinear oscillations (Beek & Beek, 1988). When the hand loop trajectory is viewed as an oscillatory system, $T_L$ and $T_U$ constitute two distinct repetitive subtasks that are repeatedly embed-

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ded in each loop. $T_L$ and $T_U$ are dynamically different in that during $T_L$ the total mass moved consists of ball and hand, whereas in $T_U$ it is only the hand. As frequency of oscillation is mainly determined by mass and stiffness, it follows that during the loaded part of the loop the stiffness parameter must change its value. Thus, this parameter modulation constitutes a shorter-term cycle which is embedded within the total cycle time of the hand loop. This behavior can be captured in an oscillator model, which provides for a parameter modulation embedded in the overall cycling frequency of the total system. The stability of such a system depends on the ratio of the frequencies of the oscillator and the parameter. For the case at hand, stable motion results from the frequency locking between the natural frequency of the hand loop and the frequency of the parameter modulation. Formal analysis predicts that stability of oscillatory system is selective of a ratio of $k = 3/4$.

**Experimental Results**

The prediction of $k = 3/4$ for stable performance was verified in a series of experiments with expert jugglers (Beek, 1989a; Beek & Turvey, 1992). Performing a three-ball cascade, jugglers produced dwell-ratios $k$ in a range from .54 to .86, with three most frequent values occurring at .61, .69 and .75, i.e., close to $k$-ratios of $5/8$, $2/3$ and the predicted $3/4$. If the number of juggled objects was increased, e.g., to $N = 5$ and $N = 7$, jugglers were increasingly less variable and produced $k$-values that clustered unimodally at the ratio of .75. While this result followed intuition in that the freedom to vary is increasingly diminished for $N > 3$, the fact that jugglers increasingly reproduced $k$-values of $3/4$ is consistent with the prediction. Moreover, an experiment where jugglers juggled balls of different weights showed that $k$ was insensitive to kinetic variations. This insensitivity is implied in and corroborates the formulation of constraints in entirely kinematic terms. An unpredicted and yet unexplained result in juggling $N = 3$ was that $k$ systematically increased with hand loop period $T_H$. This systematic result, significant for $N = 3$ and $5$, was further tested by having subjects juggle scarves, because these light-weight objects lengthen the juggling cycle periods considerably. Across the entire range of juggling frequencies the same dependency of $k$ was observed.

**Rhythmic Bouncing of a Ball**

Let's make a transition and move to simply bounce a ball with a racket. How can ball bouncing be understood from a dynamic perspective? A direct transfer of the juggling strategy appears impossible at first sight as the preceding analysis focussed exactly on that element that is absent in ball bouncing: an extended temporal interval of approximately 200 ms during which the juggled object is in contact with the hand. In contrast, the time of impact in bouncing is negligibly short to be of any plausible consequence for control
(approximately 20 ms, depending on the elasticity of the racket's surface). However, when the continuous hand motions in juggling were interpreted as nonlinear oscillators and stability considerations were brought forth to ground predictions about the selected values of \( k \), the strategies begin to converge.

The starting point for a formal analysis of the bouncing task is to reduce or abstract the task to its central components: The racket-effector movements are modeled as a planar surface performing periodic vertical motions by which a ball is repeatedly bounced in the air (Figure 1B). To keep the model mathematically tractable, the trajectories of surface and ball are confined to one vertical dimension. With the minimal assumptions of periodic motion of the surface, the laws of ballistic flight and inelastic impact for the ball, equations of motion can be written in the form of a discrete map which relates the successive impacts at time \( n \) and \( n+1 \) (Schaal, Sternad, & Atkeson, 1996; Sternad, Schaal, & Atkeson, 1995). (*) This so-called impact map is sufficient for the questions asked because the ball trajectories between impacts only follow basic mechanic laws and the periodic racket movements between impacts are of no direct consequence for the ball trajectories. This model has been studied in various textbooks on nonlinear dynamics as it shows all classical properties of a nonlinear system, such as fixed point and periodic attractors, quasiperiodicity and period-doubling route to chaos, similar to the well-known logistic map (Guckenheimer & Holmes, 1983; Tufillaro, Abbott, & Reilly, 1992). The model only contains two parameters: \( \alpha \) which is the coefficient of restitution of the ball, determining the amount of dissipated energy at impact, and \( g \) which is the gravitational constant. Although the equations cannot be solved analytically, stability analyses can be performed. Stability analysis, which is a central tool in the analysis of nonlinear systems, searches for conditions for which the state variables at time \( n \) are identical to time \( n+1 \), i.e., the system does not change. Expressed in the language of the movement task, conditions are sought for which the ball’s trajectory remains invariant and the performance remains entirely rhythmic. An important feature of such stable solutions is that they are an attractor, which implies that when the system is in this regime any perturbations «passively» converge back to the attractive regime. This strategy stands in contrast to the classical approach of control theory in which a perturbation of the ball would be compensated for by an explicit change of the actuator trajectory.

A central result of this stability analysis is that the acceleration of the racket at impact \( \ddot{x}_R \) completely determines the stability properties of the ball’s trajectory. Specification at impact

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trajectory. Specifically, stable solutions are defined when the racket's acceleration at impact $\ddot{x}_R$ is negative in the range $\ddot{x}_R \in [-10.0 \text{ m/s}^2]$.\(^{(\text{**})}\)

The central lesson from this analysis is that $\ddot{x}_R$ is the major variable which determines the stability of the solution. Admittedly, though, these conditions are not very stringent. Hence, in addition, a second stability analysis was performed in order to investigate whether the degree of stability across this wide range is different. Essentially, the model was simulated for different $\ddot{x}_R$ values and the relaxation transient from different initial conditions towards the attractor was numerically estimated (for details see Schaal, Sternaud, & Atkeson, 1996). As dynamical stability is closely related to variability, this numerically derived stability index serves as an inverse index for performance variability of the juggling trials. Figure 3 shows the result of this analysis by the solid line. Note that while the $\ddot{x}_R$ values corresponded to the values of $\dddot{x}_R$ at which the system was analyzed, the amplitude values in arbitrary units and were scaled to the amplitude of the data. In sum, the two analyses led to two major predictions: (1) Dynamically stable performance is obtained if $\ddot{x}_R$ satisfies (for $a = .71$): $\ddot{x}_R \in [-10.0 \text{ m/s}^2]$. (2) The degree of stability is a nonlinear function of $\dddot{x}_R$, where the approximate range of $\dddot{x}_R \in [-3.5, -1.5 \text{ m/s}^2]$ predicts the highest degree of stability, or lowest variability.

**Experimental Results**

A first study by Schaal et al. (1996) confirmed these predictions. The experiment was conducted with a special apparatus built to be in close correspondence to the model. Subjects bounced a ball rhythmically by moving a handle of a 1-m long lever arm with a racket attached at the other end. A pantograph linkage ensured that the racket's surface remained strictly horizontal. The ball was affixed to a 1-m long boom to confine its trajectory to a (curvilinear) linear path. Hand and racket as well as ball trajectories were thereby strictly confined to one dimension. Six subjects were instructed to hit a ball rhythmically such that the ball amplitude was invariant over repeated impacts throughout the 30-second-long trials, a pattern corresponding to a stable period-one solution of the model. They performed trials at three different ball amplitudes: preferred, low and high. Additionally, the apparatus allowed the manipulation of «gravity» by attaching a counterweight at the end of the boom which effectively slowed the ball's acceleration to $g_R = 7.0 \text{ m/s}^2$. To test the two predictions, mean and standard deviations of $\dddot{x}_R$ over the approximately 30 impacts during one trial served as empirical equivalents for the predicted acceleration and degree of stability, respectively. Figure 3A shows that the average $\dddot{x}_R$ of individual subjects varied between -54 m/s² and -8.94 m/s² but individual subjects clustered around different preferred values (see Figure 3B). The overall average $\dddot{x}_R$ across all trials and subjects was -3.44 m/s². This result is noteworthy because the energetically more

\(^{(\text{**})}\) The lower limit of the range depends on the two parameters in the model, $a$ and $g$. For the conditions used in the present experiments, -10 m/s² is a good approximation.
Fig. 3. - Standard deviations of the acceleration at impact calculated for each trial, SD($\ddot{x}_R$), versus the means of acceleration at impact calculated for each trial, $\ddot{x}_R$. The solid line represents the predicted degree of stability as calculated by the numerical stability analysis (the vertical axis for the simulation results are in arbitrary units and are scaled to fit the values of the data). (A) Experiment 1: $\alpha = .71$. (B) Experiment 2: $\alpha = .39$. Note that the different coefficients of restitution yield differences in the predicted stability.
economical solution would be to hit the ball at maximum velocity or $\dot{z}_k = 0 \text{ m/s}^2$. The important gain of this dynamically stable strategy is that small inevitable perturbations need no active corrections as the coordinative regime is attractive and perturbations will «passively» converge back to the stable trajectory. This contrasts to a dynamically unstable solution where every deviation from the trajectory requires active correction. The latter strategy is an «expensive» strategy which is also likely to lead to more variable solutions. That this solution is also possible is shown by some trials where $\ddot{z}_k > 0$.

The second prediction was tested by analyzing the variability of the performance, quantified in the standard deviations SD($\ddot{z}_k$). To compare it with the numerical stability predictions, it was of interest whether different values of $\ddot{z}_k$ were accompanied by different SD($\ddot{z}_k$). Figure 3A shows the means of $\ddot{z}_k$ per trial for all 6 subjects plotted against SD($\ddot{z}_k$). As predicted, the majority of subjects' trials $\ddot{z}_k$ fell into the restricted range of maximal stability and were accompanied by less variability.

Yet, the task and the movements were highly constrained and had little resemblance with the real-world task of bouncing a ball with a racket. This raised the question whether in less constrained settings these results will still be upheld. Testing this robustness was the goal in a follow-up experiment where the constraints for arm movements were relaxed (Sternad, Katsumata, Duarte, & Schaal, 1999; Sternad, Duarte, Katsumata, & Schaal, submitted).

Six subjects performed ball bouncing with a hand-held racket in the air such that the ball amplitude was invariant over repeated impacts. Again, the ball was affixed to a long boom to avoid loosing the ball and to provide a conservative transition from the one-dimensional model to more unconstrained movements. To additionally test the effect of the model parameters, a different ball with $a = .39$ was used. Note, that although the movements of the subject's hand and arm were unconstrained, the ball's trajectory was still confined to the vertical dimension and it was sufficient to measure only the racket's vertical displacement and acceleration. Figure 3B shows the median values of $\ddot{z}_k$ for all trials and all subjects plotted against the range of $\ddot{z}_k$. Again, the subjects predominantly preferred movement solutions with negative $\ddot{z}_k$ (see also the inserted histogram). Note also, some positive $\ddot{z}_k$ demonstrate that the choice of $\ddot{z}_k < 0$ is not trivial. Additionally, the variability associated with the different $\ddot{z}_k$ differed in those values in the approximate interval of $\ddot{z}_k \in [-6, -2 \text{ m/s}^2]$ and was associated with the lowest range of $\ddot{z}_k$ (note the predicted curve is different due to a different $a$). Different symbols denote different subjects showing that each subject clustered around preferred values. Across subjects, though, $\ddot{z}_k$ values covered a large portion of the predicted stable interval. These results corroborated the results from the previous study. Note though, that the actual movements were less tightly controlled and no longer bound to the model's one-dimensional motions.

While the summaries on juggling and ball bouncing evidently reflect different formal approaches, there are also striking similarities in strategy which, in fact, exemplify the essence of a dynamic systems approach to movement. What matters first and foremost, is the identification of stability and constraints of the task. Shannon's equation for juggling relates the essential temporal quantities and thereby sets restrictions on the free variation of these temporal quantities. Within these constraints, though, an infinite number of realizations is possible. These realizations are shown to be best captured in the compound variable $\dot{k}$, the relative time that a ball is held within one hand.
cycle. In ball bouncing, the first (local linear) stability analysis determines the boundaries for dynamically stable solutions which are defined for the variable $\bar{x}_r$. But the boundaries are wide and still allow for an infinite number of realizations. Therefore, further rationale is needed to define subsets of «preferred» values, i.e., values that are more stable than others. While this differentiation was obtained through a numerical stability analysis resulting in the prediction of different variability for subregions of the stable range, similar predictions were brought forth in juggling, based on different winding numbers that corresponded to different dwell ratios $k$. Although Beek and colleagues did not quantify different levels of variability for different $k$, the argument carried the same logic. It is feasible to predict that deviations from these ratios are more likely for higher winding numbers, as they have smaller basins of attraction. Consequently, a higher degree of variability could be expected. To what extent this is the case needs to be relegated to future study.

**Interpretations of Learning**

Having gained a formal understanding of the task, which provided empirically valid predictions, questions about the process of learning and stages in acquisition come within answerable reach. As will be exemplified below, theoretically grounded compound variables can serve as the task-relevant reference frame in which the acquisition process can be measured. To date, questions on skill acquisition have been primarily addressed by theories of a more cognitive slant. The predominant view is that the learning process is either one of intellectually constructing motor programs (Austin, 1976), or of developing grammatical rules that then make up a cognitive representation or an abstract control structure (Norman, 1976; Schmidt, 1975; Schmidt & Lee, 1998). Dynamic systems theory, on the other hand, has made relatively few inroads into formulating the process of learning. One of those views is grounded in the synergistic approach and has claimed that the acquisition of a new pattern should be understood as the transition from the intrinsic dynamics of a particular task to the desired one which is specified by behavioral information (Schöner & Kelso, 1988a,b; Zanone & Kelso, 1992). Initial (intrinsic) state and (extrinsic) goal state are cast into the same dynamic «language» and can therefore be understood as competing and cooperating forces bringing about the transition. Despite its formal clarity, this approach is restricted to situations with well-defined starting and end states, a situation that is not necessarily typical for common learning scenarios. The example of juggling immediately shows that learning proceeds from an undefined, unor-
ordered initial state to a highly ordered one. An alternative approach, also rooted in dynamic systems concepts, has been suggested by Kugler (1986) and Newell and colleagues (Newell, Kugler, van Emmerik, & McDonald, 1989; Newell, 1991) who propose that learning is a search process which explores the perceptual-motor workspace. A problem for investigation, though, is that any investigation following this view requires a priori knowledge of this search space in order to be able to trace the exploratory search. Additionally, it is reasonable to expect that the search space itself is nonstationary and changes over practice, as the attractor layout for complex actions results from the combination of organismic, task and environmental constraints (Newell, 1986). Therefore, Beek and van Santvoord (1992) pursued a third alternative. Essential for the uncovering of the principles of skill development is to know what defines expert performance. Once, essential variables about expert performance are known, they provide a reference frame within which the learning process can be related to. Expert, or better stable performance, was defined for both juggling and ball bouncing. It will be the marker around which the evolution of the skill can be traced.

THREE STAGES OF LEARNING IN JUGGLING

From the formal analysis of juggling, it is straightforward that the invariant \( k \) can play the role of a reference scale. If small winding numbers, or \( k \) ratios composed of small integers, characterize stable juggling, then the transition from unskilled to skilled performance should be reflected in significant changes in this ensemble variable. Beek and van Santvoord (1992) delineate three stages of learning: 1) The basic frequency relations between hands and balls should be developed, which is tantamount to actually keeping balls in the air. 2) Acquisitional changes should occur in the internal partitioning of component times where \( k \) should develop towards the most stable ratio of 3/4. 3) On the basis of prior results on professional performers, which showed that experts tend to deviate from the exact ratio of 3/4, it is argued that a third stage exists in which higher-order mode-locks are developed, leading to \( k \)-values that are smaller than .75. This pattern is associated with juggling with room for artistic interpretation.

Experimental Results

In a two-week-long learning study 20 novices were guided through a controlled learning process in 10 sessions. Focus of analysis was the identification of the three stages in terms of the time evolution of \( k \). After three sessions, the first stage of learning was accomplished and
subjects mastered on average 3.4 complete cycles. This initial stage was traced by using a deviation measure for individual hand/ball cycle times, in accordance with mastering the real time demands of a successful juggle (not to drop the balls). After this initial stage, the focus turned to inspect whether $k$ would be fine-tuned towards the theoretically predicted value. However, even after only 3 practice sessions the overall subject average value for $k$ was already extremely close to .75. To probe further whether differences in skill level were associated with differences in $k$, the subject pool was divided into slow and fast learners based on performance in the first three sessions. Indeed, nuanced but significant differences were revealed: while slow learners started out with $k = .79$ and changed towards the critical ratio in the remaining 7 sessions, the fast learners had already achieved $k = .75$ after the 3 sessions, but further refined their patterns towards mean $k$ values of .73. Even though these changes in absolute values appear very small, they were statistically significant and underscored the sensitivity of this ensemble variable. These results provided evidence that the second stage of learning was indeed signified by establishing the stable ratio. Evidence for the third stage, in which jugglers are able to deviate and «play» with the temporal constraints, is only hinted at in the small changes in $k$ (.02) for fast learners. Comparison with previous results of professionals, where $k$ was as low as .54, this is only a trend. Given the short period of practice, though, this is not surprising. In sum, $k$ was confirmed as an appropriate ensemble variable around which learning can be traced.

**Attunement to Dynamic Stability in Ball Bouncing**

While cascade juggling is a complex task and requires many hours of practice in order to safely keep three balls in the air, not to mention the years of dedicated practice that performing artists have undergone, ball bouncing is comparatively simple. Especially in the highly constrained apparatus in which the ball cannot be lost and unskilled behavior can only manifest itself in ball amplitude variations. Should one therefore drop the question of learning to bounce as pointless? Not at all, given the many different stable realizations which are only bounded by $-10 < \dot{z}_R < 0 \text{ m/s}^2$, refinements towards maximally stable regions of $\ddot{z}_R$ are nevertheless to be expected. Noticeable changes of this refinement process, though, probably occur over a shorter time scale. Therefore, no practice sessions were administered and subjects performed experimental trials after a very brief time of familiarizing (5 min) with the apparatus. The question asked is whether and how subjects, when confronted with this task for the first time, attune to the predicted maximally stable performance. If so, do they change their performance pattern continuously towards the predicted range of solutions?

**Experimental Results**

To investigate this short-term learning or attunement, subjects performed the experimental task with unconstrained arm movements. This analysis was done as part of Experiment 2, already reported, to get three repetitions of 150 cycles, averaging this figure in 20 separate trials. This resulted in a mean error of $\pm 0.1$ for all trials. The table below shows the results of these trials.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Average Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>0.03</td>
</tr>
<tr>
<td>3</td>
<td>0.04</td>
</tr>
</tbody>
</table>

**Individual Differences**

Thus far, the pursuit of testing the relevance of the findings in the latter issue to the individual preferences when a teacher is working with a patient, 

<table>
<thead>
<tr>
<th>Framer Acceleration (m/s^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>0.8</td>
</tr>
<tr>
<td>1.0</td>
</tr>
<tr>
<td>1.2</td>
</tr>
<tr>
<td>1.4</td>
</tr>
</tbody>
</table>

Fig. 4. - Means and (Experiment 2), Trial 1. For all three discernable. This utility (see Figure 3).
2, already reported above (Sternad, Katsumata, Duarte, & Schaal, 1999). The experiment contained three repetitions per amplitude condition, which were presented in random order. The average results in Figure 4 illustrate that the $\bar{x}_\alpha$ values showed a significant decrease towards more negative values. This main effect held across all three amplitudes conditions. The standard deviations of $\bar{x}_\alpha$ decreased similarly but only for intermediate and high ball amplitudes. Thus, the limited learning process across the three trials demonstrated that subjects progressively attuned towards a performance variant, which offered higher dynamic stability. This supported the pivotal role of dynamic stability in subjects’ performance.

![Graph showing mean and standard errors of all trials and subjects in the three amplitude conditions (Experiment 2). The three adjacent bars represent the three trials performed in the same condition. For all three ball amplitudes the trend to decrease $\bar{x}_\alpha$ towards more negative values is discernable. This trend is towards $\bar{x}_\alpha$ values which are associated with a higher degree of stability (see Figure 3).](image)

**Individual Differences**

Thus far, the experimental results mainly reported subject averages in pursuit of testing the predictions and, more fundamentally, corroborating the relevance and usefulness of the identified variables. Individual differences undoubtedly existed but they were subsumed into overall significant trends. While this statistical approach is obviously necessary and widely used in behavioral sciences, it smooths over differences between individuals. Yet, the latter issue is of tantamount importance for the understanding of movement coordination. This is evident especially in practical situations, such as when a teacher is confronted with students of different ability, or a therapist with a patient, and judging their behavior against uniform templates speaks
against all reason. But in most cases it has proven difficult to go beyond simply accepting the seemingly arbitrary skill levels and skill interpretations. There is an urgent need for a principled route to understanding variant behaviors and individual differences. What is suggested below, is that the formal approach to both juggling and ball bouncing proves to be a useful framework for the understanding of seemingly qualitative differences.

SKILL AND ARTISTIC FLAIR IN JUGGLING

Beek’s original experiments with expert jugglers performing a three-ball cascade showed almost discouraging results with respect to the prediction of \( k = 3/4 \). The temporal pattern of four juggling artists gave \( k \)-values that ranged between .54 and .83, with the mean at \( k = .71 \). Why should experts not lock right into the most stable regime? The theoretically predicted result was only obtained in juggling five and seven balls where the spatiotemporal constraints were a lot tighter and allowed less room for individual interpretation (Beek, 1992). An interesting interpretation of these results is the hypothesis that skilled behavior is not situated right at the most stable regime but rather resides at the ‘margins’ of stable regimes. While stability constitutes a reference point for the mastering of a skill, it does not allow sufficient flexibility and room for ‘surprises’. Adaptability, the hallmark of expert performance, is only obtained at the boundaries of stability, where quick transitions and fleeting deviations from predicatable behavior are possible. A vague correspondence can be drawn with the thesis that biological systems evolve towards the ‘edge of chaos’. What is meant by this phrase introduced by Kauffman (1993) is that biological systems tend to parameterize their systems close to chaotic regimes as these regimes allow a far richer and more flexible repertoire of behavior. The \( k \)-values found in juggling performers with long years of practice correspond to higher winding numbers, i.e., \( k = .54 \) corresponds to \( W = 12 \). Such higher winding numbers have a smaller basin of attraction, and deviations from the exact winding number lead the system more readily into a chaotic regime.

MOTOR EQUIVALENCE AND TOPOLOGICAL ORBITAL EQUIVALENCE IN BALL BOUNCING

For ball bouncing a route to understanding individual differences has come forth directly from the analysis of topological equivalence of the model. Topological orbital equivalence is a central concept in the theory of nonlinear

dynamics, wi
dynamics, which formalizes criteria for which one dynamical system can be continuously transformed into another. As such, it is a close formal analogue to the empirical notion of motor equivalence by capturing invariance over transformations. A scaling relation was extracted which formulates such a criterion which identifies equivalent movement realizations (Schaal, Sternad & Arkeson, 1996; Sternad, 1998). If the amplitude of the racket’s trajectory scales with the squared period between two successive impacts between ball and racket, then different spatiotemporal solutions as displayed by different individuals can be regarded as variants of the same underlying dynamic regime.

Experimental Results

The data of Experiment 1 served for the evaluation of this scaling relation. Important for this analysis is that all three amplitude conditions were additionally performed under two gravity condition, $g_N$ (normal gravity) and $g_R$ (reduced gravity). First, the amplitudes and periods of the racket movements were determined. Then, the average amplitude per trial was regressed against the mean squared period. The regression obtained $r^2$ values between 92 and 99%. Figures 5A, B and C show the results for three subjects. Note that the regressions were performed separately for $g_N$ and $g_R$. These results illustrate that different trials of one subject performed at different amplitudes obeyed a constant scaling and can be regarded as topologically equivalent. Racket trajectories from repeated performances of one subject may vary in their kinematic characteristics, specifically in their amplitudes and, hence, their periods, but they do not require different underlying dynamic regimes and can hence be interpreted as satisfying motor equivalence.

Despite the fact that subjects display invariant spatiotemporal relationships across different bouncing heights, the individual subjects visibly display differences in the actual slopes of the linear fits. Do these differences finally reflect the unexplainable individual «choice», or can they be accounted for within the dynamic model? As already reported above, the individual subjects have individual «preferences» for $\tilde{z}_R$ values within the range of stable solutions. A simulation of the model shows that for different values of $\tilde{z}_R$ the linear scaling relationship results in linear relationships with different slopes (see for more detail Schaal, Sternad, & Arkeson, 1996). Moreover, this simulation shows that for each of the six values of $\tilde{z}_R$ the two gravity constants produce pairs of slopes that form different angles with each other (the solid line represents $g_N$, the dashed line is $g_R$). As shown in Figure 5D, the difference in slopes for $\tilde{z}_R = -1 m/s^2$ for $g_N$ and $g_R$ becomes progressively smaller towards $\tilde{z}_R = -6 m/s^2$. Comparing the individual subjects’ $\tilde{z}_R$ and their slope deviations between $g_N$ and $g_R$ shows that, indeed, this individual instantiation of the scaling relation is directly constrained by their choice of $\tilde{z}_R$.

To sum, the model provides a basis that explains variations between individuals in their kinematic characteristics. An individual choice of the racket’s acceleration at impact, the critical variable $\tilde{z}_R$, which is itself within the stable range as determined by the dynamic model, further determines variations over different instantiations. However, these variations are still highly constrained and can be shown to belong to the same underlying dynamic regime.

479
Fig. 5. - Panel A, B, C: Three subjects and their scaling relation between the racket’s amplitude, $\tilde{x}_R$, and the squared period of racket movements, $t^2$. Panel D: Results of a simulation of the model equation performed for racket accelerations from $-1$ to $-6\,\text{m/s}^2$. Analyzing amplitude and period of racket motions, different $\tilde{x}_R$ lead to different scaling relations for the two gravity conditions.

The Role of Perceptual Information

Having gained some insight into formative principles and constraints of the task, a solid basis is prepared from which one can question what perceptual information is necessary to attune to stability as characterized by specific values of the quantities $\epsilon$ or $\tilde{x}_R$. The critical variable can now constitute an orientation post around which the effect of perceptual manipulations can be gauged. In both juggling and ball bouncing, principal sources of information are undoubtedly the haptic and visual modalities. But to what degree is either modality involved? How does the stable performance break down when subjects are partially deprived of such information?

Experimental Re.

With the artificial flight trajectories, even after the minimal perturbations, the subject’s perception changed. The trajectory showed the ball trajectory pattern, whether action is performed by the subject’s eye, the hand, the arm, the leg, or the head, the trajectory pattern remained the same. When the previous trajectory was altered, the subject’s perception changed. The two trajectories showed that perception was the same.
VISUAL INFORMATION ABOUT THE BALLS’ TRAJECTORY IN JUGGLING

In cascade juggling the visual tracking of the thrown balls appears essential, get proficient jugglers can juggle three balls blindly and therefore entirely rely on haptic information in their force and timing control. This extraordinary display of skill aside, for regular jugglers the immediate question is what kind of visual information do actors need? As the number of airborne objects is always greater than one, can it be conjectured that there is a specific section of the flight trajectory that one should attend to which holds all the necessary information for a successful throw and catch?

Experimental Results

With the aid of LC glasses van Santvoord and Beek (1994) manipulated the visibility of ball flight trajectories. Given an average time of 340 ms between successive ball tosses, viewing times were periodically blocked out (200 ms open vs 140 ms closed). With the question to determine the minimally necessary viewing times (for four intermediate jugglers), this viewing interval was decreased in steps of 8 ms to whatever interval the participants could maintain the pattern. A second even more interesting question was whether the jugglers would reorganize the timing of their hands in order to phase their juggling pattern so that specific segments of the ball trajectory would remain visible. This manipulation allowed the interesting issue whether action is modified in order to attain optimal perceptual information. From a dynamic perspective, the issue is to what extent there is phase locking between the viewing time and the hand cycle. Would jugglers prefer specific phase relations and would these preferred relations be re-established after perturbation?

Minimal viewing times before jugglers lost the pattern varied between 46 and 81 ms, which is less than 10% of the ball flight time. Next, attention was directed to the phase relation obtained between viewing and the hands’ movements. Unexpectedly, only one out of three jugglers established stable phase locks. Analyzing the preferred portion of the ball flight’s interval showed that the segment following the zenith of the trajectory appeared to be the preferred interval carrying anticipatory information about the ball catch. This runs counter to common advice in teaching juggling, which is: Look at the highest point and throw the next ball when the previous one reaches the top.

The two other jugglers drifted in their juggling frequency so that different parts of the balls’ trajectories were visible in the course of one trial.

Assuming that this lack of phase locking may have been due to the unfamiliarity of the experimental situation, the same participants underwent a series of practice sessions. Surprisingly, even after eight 30-min practice sessions, none of the subjects obtained stable phase locks. Apparently, the perceptual cycle and action cycle did not demand to be in strict phase relationship. This showed that functional information is contained in many segments of the trajectory and is not confined to one critical interval. However, analysis of the variability of the kinematic trajectories of balls revealed that the spatial and temporal variability of key events, such as the position of ball release and catch or the relative phase between left and right hand, was significantly smaller when phase locking was present. The organizing role of stable phase locking was once again corroborated in experiments where viewing intervals were perturbed.
An unexpected lengthening of the viewing interval showed that previous phase locking was reestablished after the perturbation.

To explore the contribution of haptic versus optic information, juggling was performed by a pair of jugglers. This modification of the skill implies that when one juggler throws the ball, and the partner catches it, the partner has no haptic information about the ball’s trajectory and has to entirely rely on visual information. Using the same conditions for shortening viewing times as before, minimal viewing times were found to be identical to individual juggling. This finding led to the conclusion that haptic information plays a minor role.

Besides the only tentative support for the hypothesis about phase locking between hand movements and perceived ball trajectory, what remains a result in accordance with the theory of phase locking was that variability increased in the absence of phase locking. Dynamic stability is not defined for the action alone but is the result of the action-perception system. This tight connection between variability and stability is also the hallmark of a dynamic approach and will be revisited when perceptual manipulations are applied to ball bouncing.

**Haptic and Visual Information in Ball Bouncing**

*Experimental results.*

Using the same apparatus as in Experiment 1, three subjects were instructed to bounce a ball rhythmically with a steady ball amplitude (Sternad, Duarte, Katsumata, & Schaal, submitted). Three perceptual conditions were imposed: subjects closed their eyes and thereby excluded visual information (*no-VI*), haptic information about the impact was eliminated (*no-HI*), a control condition with full perceptual information was performed (*FI*). To exclude haptic information, a telerobotic device was attached to the ‘juggling arm’ that subjects moved, which recorded its angular displacements. This signal served as the desired trajectory for a ‘robot’ to move the actual racket. In this way, subjects moved the handle but did not get any tactile information about the ball contact. At the same time they saw the continuous trajectory of the ball which was bounced by the robot. All three conditions were performed with three different ball amplitudes for 30s, with two repetitions for each condition.

Exemplary time series of ball and racket displacements and the corresponding phase portraits are shown in Figure 6 for the three different perceptual conditions. Quantitative comparison of the three perceptual conditions revealed significant differences in $\tilde{X}$ and its variability across the duration of one trial (30 sec). Median and range of $\tilde{X}$, calculated per trial (the data were not normally distributed), served as empirical estimates to test the model’s predictions statistically. The *FI* condition (Figure 6A) showed a tight cluster of $\tilde{X}$ around $\tilde{X}= -4.16 \text{ m/s}^2$ and signified a dynamically stable solution, replicating previous results. The *no-VI* condition (Figure 6B) was characterized by a median $\tilde{X}$ value of $-4.43 \text{ m/s}^2$, but this predominantly stable solution was significantly less consistent. Note, that it was only for the very short moment of contact that kinetic information about the impact was available. In contrast,
when continuous kinematic information about the ball's trajectory throughout the entire cycle was available, but no kinetic information (Figure 6C), $\ddot{x}_k$ frequently scattered into the positive range (median $\ddot{x}_k = -0.31 \text{ m/s}^2$). Its trial variability, on the other hand, was unexpectedly low. Figure 7 shows all trial means together with the individual subject averages for three perceptual conditions, respectively. The triangular connections for each subject highlight that all subjects showed the same relational pattern. Analysis of variance verified that median $\ddot{x}_k$ was different for the three conditions (no-VI < FI < no-HI; $p < .05$) and that the range of $\ddot{x}_k$ was highest in no-VI, while not different for FI and no-HI. This latter finding ran counter to the predictions. When only visual information was accessible (no-HI), many trials were performed with $\ddot{x}_k > 0$, indicating dynamically unstable solutions. However, variability remained low and therefore suggested that subjects employed a different strategy that corrected potential errors resulting from perturbations. In conclusion, when subjects were deprived of either kinetic or kinematic information, kinetic information provided the major source of information for parameterizing the movement system to this stable regime. When kinematic information was the principal source subjects tended to fall back on other, most probably feedback based error correction control strategies.
Identifying Constraints – Summarizing the Strategy

Bouncing a ball and juggling three balls are two very different kinds of movements with different demands on the skill of the actor. And yet, both movements have in common that they involve the control of one or several balls where the impact or the toss fully determines the ball's flight trajectory. In both cases, it was shown that well defined physical conditions constrain the actor's movements for establishing an invariant rhythmic pattern. These constraints served as the basis to identify further criteria for the actor's options and, more importantly for this dynamic approach, his boundaries in executing the task. Using concepts from nonlinear dynamics, the options of the actor can be further limited, and dynamically stable solutions can be defined for a subset of movement solutions within the boundaries of possibilities. Phase locking of oscillations and dynamic stability for the periodic solution of the ball bouncing map were the concepts used for juggling and bouncing, respectively. The review traced each of the two lines of research and presented the concepts from nonlinear dynamics and experimental results in parallel. There were evident differences in the key variables used to capture the task as well as in the type of physical prin-

ciples involved and the other strategy and more one component being able to infer that selected subjects in a different move were interpreted by others and, hence, the dynamic pertinent issues...
principles invoked to provide criteria for performance stability. This contrast gave an impression of the spectrum of tools that a dynamic approach has to offer. On the other hand, the dual presentation also demonstrated the similarity in strategy and methodology that the two lines of work followed: In both approaches one compound variable became the reference frame for evaluating skilled performance and the process of learning. In both approaches, the role of perceptual information was evaluated by the decrement in stability of performance that selected perceptual manipulations induced. Individual differences between subjects in ball bouncing were shown to be the consequence of their choice of the key variable, which further constrained the kinematic properties across different movement realizations. Individual differences between expert jugglers were interpreted as their «deliberate» choice of a solution that is not tightly constrained by maximum stability but rather one that affords them more flexibility and, hence, flair. In sum, the goal was to show that an analysis from a nonlinear dynamic perspective could establish a basis from which a set of further important issues can be addressed, in a quantitative and physically principled manner.

Identifying Constraints Versus Order Parameter Dynamics

Having outlined the strategy taken in bouncing and juggling, it may have become apparent to readers familiar with the literature on the dynamic systems approach that this strategy differs considerably from the framework developed in the area of interlimb coordination, often referred to as dynamic pattern approach (Kelso, 1995). Interestingly for the present context, the very same framework has been applied to account for a ball-batting task which is very similar to the two tasks in focus (Sim, Shaw, & Turvey, 1997). Specifically, the task consisted of a bat attached to a hand-held pendulum, which was swung rhythmically to strike a ball that was suspended from a string. Although a detailed account goes beyond the scope of the present paper, a brief comparison of the theoretical approaches should be attempted. As repeatedly laid out in studies on bimanual coordination, the central first step in the dynamic pattern approach is the identification of an order parameter which captures the so-called intrinsic dynamics of the system. Relative phase between the oscillating limbs has been verified to play the role of such an order parameter for rhythmic interlimb coordination. The second step is the formulation of a potential equation for this order parameter which captures the stable states described by the invariant value(s) of this collective variable. An important extension to this approach was made when it was shown that extrinsic task demands, present in terms of task instruction or physical constraints, could be modeled as an additive term in this

485
potential function (Schöner & Kelso, 1988a, b, c). This strategic move showed that intrinsic and extrinsic dynamics could be captured at the same level of analysis. In this spirit, the task of batting a ball was modeled and relative phase was determined between the ball and bat's periodic movements (Sim, Shaw, & Turvey, 1997). While striking the ball at the preferred frequency and amplitude was captured by a potential equation representing the intrinsic dynamics, with an attractor at 90 degrees phase relationship, additional task requirements, like hitting the ball against a wall or hitting the ball with a specified amplitude, were added as additional terms, thus constituting the extrinsic dynamics. Stability analyses of the model successfully predicted stable phase relationships between ball and bat and the associated degree of stability as a function different parameters, such as task demands as well as eigenfrequency differences between pendular ball and bat. Without going into further detail, it is evident that this approach is considerably different from the one adopted for bouncing and juggling: While in ball bouncing a simple mechanical model was formulated which can generate the complete kinematic time series of a bouncing ball and racket, the dynamic pattern approach operates at a meta- or phenomenological level. The potential equation written for the collective variable relative phase only describes the stable attractor states that the system displays. These equations cannot generate kinematics in terms of state variables but rather describe the movements of a higher-order variable. A yet different level of modeling was pursued in juggling, where fundamental relations between component times provided boundaries for what movements are realizable. Additional arguments derived from the theory of phase locking specified a set of particular values as preferable for stable performance. As in the dynamic pattern perspective, no explicit generation of movement in terms of state variables was attempted. Needless to mention, all three approaches have their merits as well as shortcomings. At this still early stage in the development of dynamic systems tools for movements it remains a useful exercise to recognize strategic differences and be aware that there is a multiplicity of tools available.

Conclusions and Outlook

Let's conclude this theoretical review with a more practically oriented outlook. Can any of these insights and results be applied to other ball skills? As already mentioned in the introduction, there is a large number of ball games, which are not too remote relatives of the experimental paradigms of bouncing or juggling balls. These games can be different in the kind of ball contact, in the kind of body parts that make contact with the ball, the kinds of balls used, the number of instance, m as bouncing, the ball trajectory became longer hori- theoreti cal plicated. It want to refer to further i

Of course, hands on two's, three variants of squash, all rhythmical treatment would have prob

Acknowledgment

This wo-
number of balls involved, or in the number of hands or people involved. For instance, many children’s games involve the cyclic manipulation of a ball, such as bouncing a ball against a wall or the floor, or the kicking a soccer ball repeatedly in the air with one foot. In the latter example, the soccer ball is “bounced” with the foot similar to the studied racket task. Also similar to ball bouncing, the ball is ideally confined to a vertical flight path. In this case all predictions concerning ball control hold in equal manner. The next and more challenging question is to what degree is theoretical transfer possible when the soccer ball is kicked alternately by the right and left foot. In this case, the ball’s trajectory lies on a plane, and the impacting “surfaces”, i.e., the feet, are no longer horizontal but have to form an angle to each other. Consequently, the theoretical model and analyses have to be extended and become more complicated. In principle, methods should be generalizable but at this point I want to refrain from any predictions and leave this interesting question open to further investigations.

Of course, one variable in the family of bouncing tasks is also the number of hands or people involved, and ball games are not only played alone, but in two’s, three’s and more players. A large and important class of activities that are variants of the bouncing task are racket sports, such as tennis, table tennis, or squash, all of which engage two or more people who bounce a ball more or less rhythmically back and forth. Are any extrapolations from the present theoretical treatments possible? Again, for the model to account for, say table tennis, it would have to be extended to at least two dimensions. But a more critical problem is that in such games the task is not to establish a periodic spatially and temporally invariant pattern but rather the opposite. Players try to vary the game, such that each pass of the ball differs from the previous one. In contrast, the theoretical treatments of juggling and bouncing a ball are crucially dependent on the cyclical nature of the task. While velocity and direction of ball release determine the trajectory of the ball, any statements of stability of performance are only valid for rhythmic performance. Therefore, predictions that were derived for the acceleration at impact or the dwell ratio cannot be directly applied to a single impact or a single hand loop. With this final caveat, I hope that these few speculative comments raise the interest and may stimulate more extensions and practically oriented investigations.

Acknowledgements

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487
APPENDIX

If $x_{R,n}, x_{R,n}, \dot{x}_{R,n}, \dot{x}_{R,n}$ denote the vertical positions and velocities of ball and racket at the $n$th ball contact, the equations of motion are (Schaal, Sternad, & Atkeson, 1996; Sternad, Schaal, & Atkeson, 1995):

$$\dot{x}_{R,n+1} = -\sqrt{((1+a) \dot{x}_{R,n} - a \dot{x}_{R,n})^2 - 2g (x_{R,n+1}(t_n) - x_{R,n})}$$

$x_{R,n+1} = x_{R,n+1}$

$t_n$ result from:

$$-0.5g t_n^2 + ((1+a) \dot{x}_{R,n} - a \dot{x}_{R,n}) t_n - (x_{R,n+1}(t_n) - x_{R,n}) = 0$$

$t_n$ denotes the times at successive impacts, $a$ is the coefficient of restitution of the ball, determining the amount of dissipated energy at impact, and $g$ is the gravitational constant.

Although the equations are nonlinear, transcendental and in implicit form, and cannot be solved analytically, stability analyses can be performed.

REFERENCES


