Vacuum Selection from Cosmology on Networks of String Geometries

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Based on

1711.06685: Jonathan Carifio, William J. Cunningham, James Halverson, Dmitri Krioukov, CL, and Brent D. Nelson

1706.02299: James Halverson, CL, and Benjamin Sung

1710.09374: James Halverson, CL, and Benjamin Sung

Workshop on Data Science and String Theory

Outline

- The string landscape, bubbles, and networks.
- Geometric networks in string theory
- Concrete networks of geometries.
- Examples of geometry selection.

The String Landscape, Bubbles, and Networks

The string landscape

- There is a vast landscape of vacua in string theory; thought to be very large $> 10^{500}$ ($10^{755}, 10^{3000}, 10^{272,000}$)

  Ashok, Denef, Douglas

  Halverson, CL, Sung
  Taylor, Wang

- These vacua arise from compactifying 10/11d supergravity on a compact space, to yield an 4d effective field theory.

- Vast size of the landscape arises from the plethora of possible geometries of the extra compact dimensions and choices of discrete objects on the geometries.

The need for vacuum selection

- Why is our universe is selected?

  1. Standard Model might be generic, but this is not established.
     see e.g. Grassi, Halverson, Shaneson, Taylor
  2. Anthropic principle does not seem be to be the complete story.

- A (partial) explanation may be that certain vacua are selected over others, via some early dynamics in the landscape.

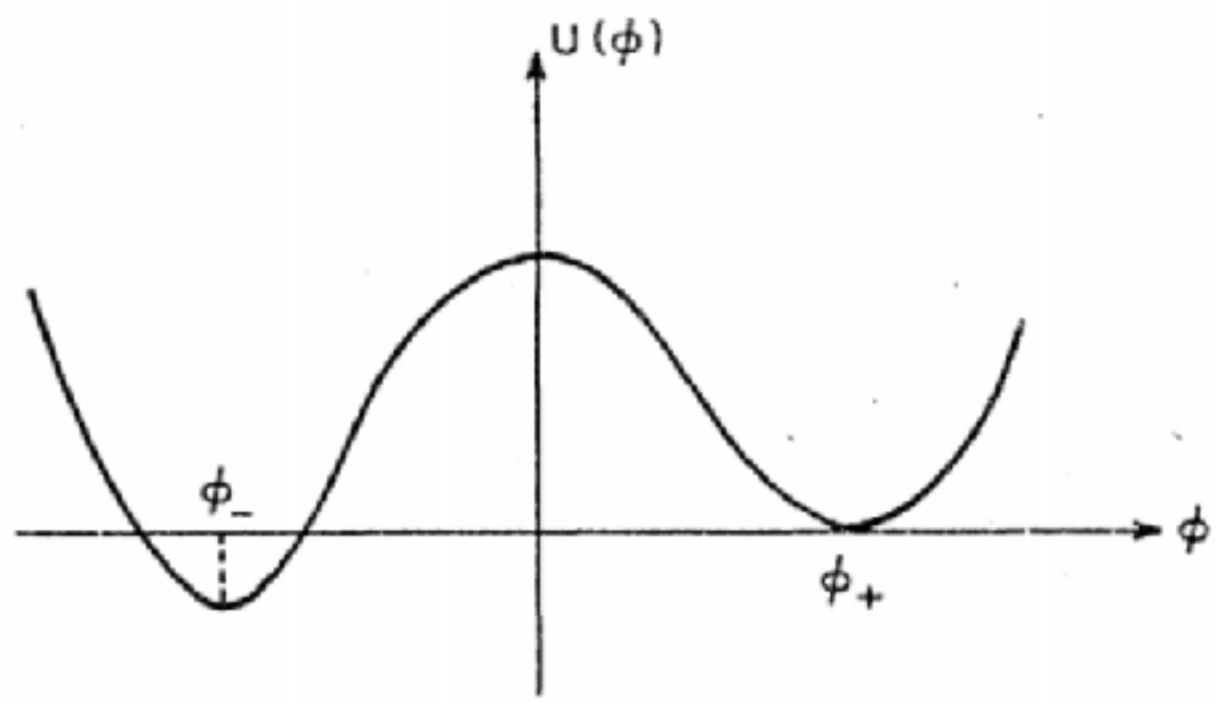
What does vacuum selection mean?

A few things we know from string theory and effective field theory:

1. There are many vacua, with varying physical properties.
2. Local vacuum transitions, known as bubble nucleation, can occur. Coleman, De Lucia
3. Bubbles can grow, collapse, nucleate other bubbles.

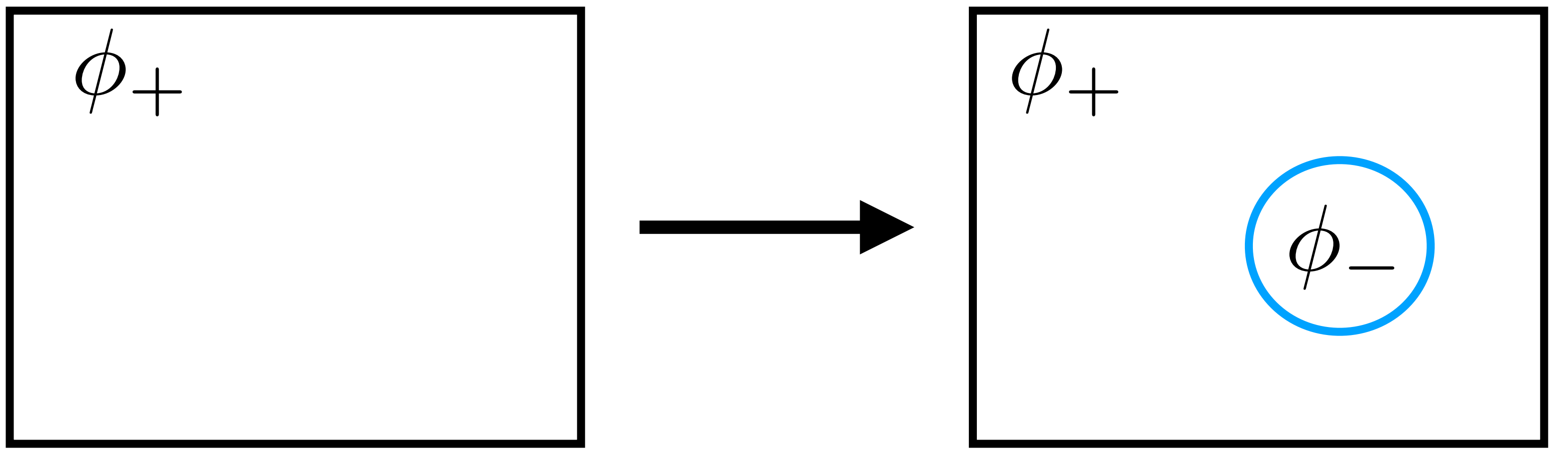
The vacuum distribution is a late-time, steady state solution of such a bubble nucleation model. Vacuum selection would be a distribution that prefers some vacua above others.

Bubble nucleation



- Original work due to Coleman and De Lucia, in a single effective field theory.

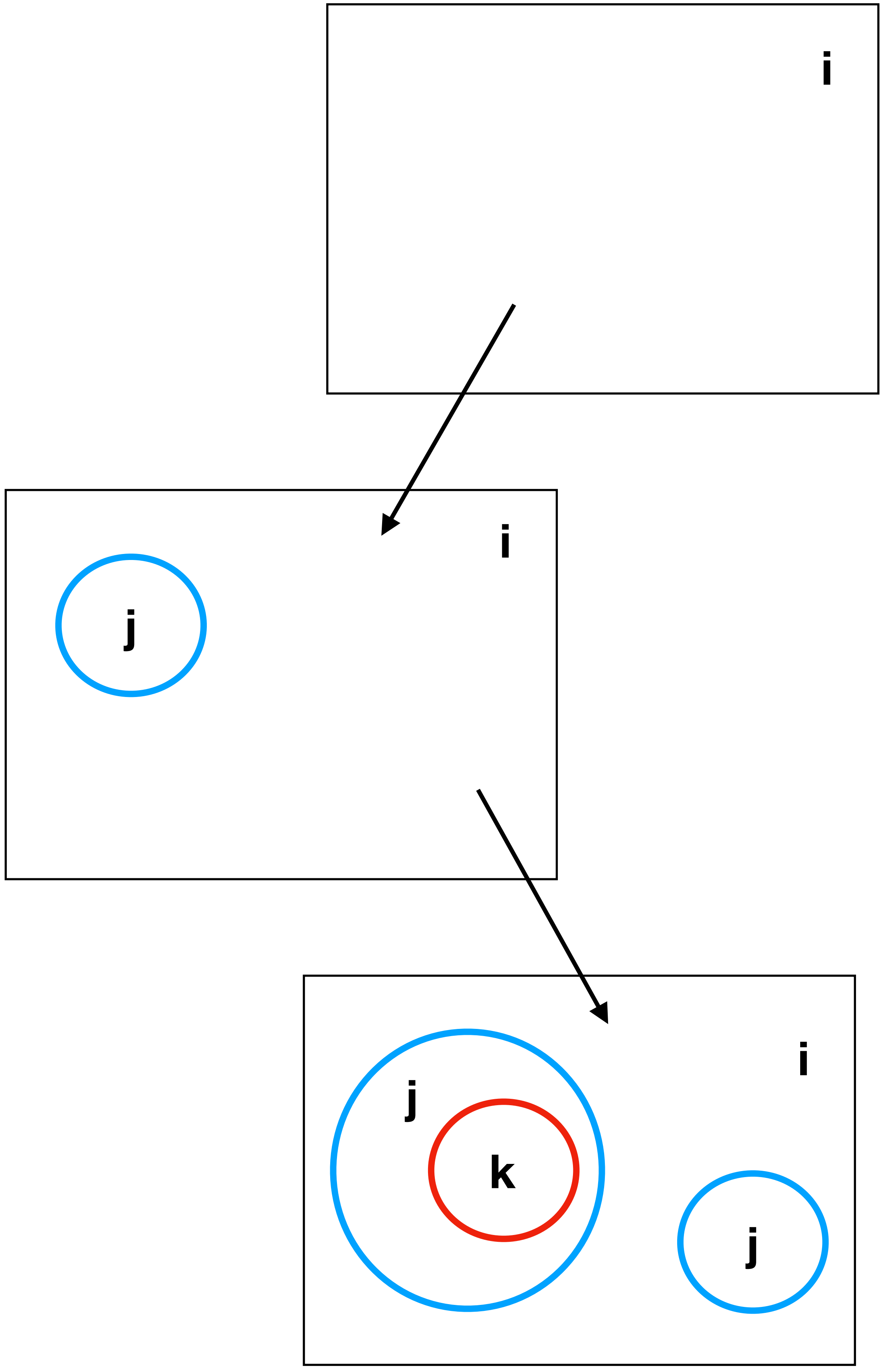
- Universe starts in false vacuum ϕ_+ , nucleates bubbles in ϕ_-



Bubble nucleation and graphs

- In general, there can be many vacua.
- Given a vacuum **i**, bubbles in another vacuum **j** can form in local patches.
- Bubble nucleation rates Γ_{ij} from vacuum **i** to vacuum **j** depend on microphysics.
- Picture:

  1. Start in vacuum **i.**
  2. Nucleate a bubble in **j.**
  3. **j** nucleates a bubble in **k**, **i** nucleates another **j** bubble, and so on.

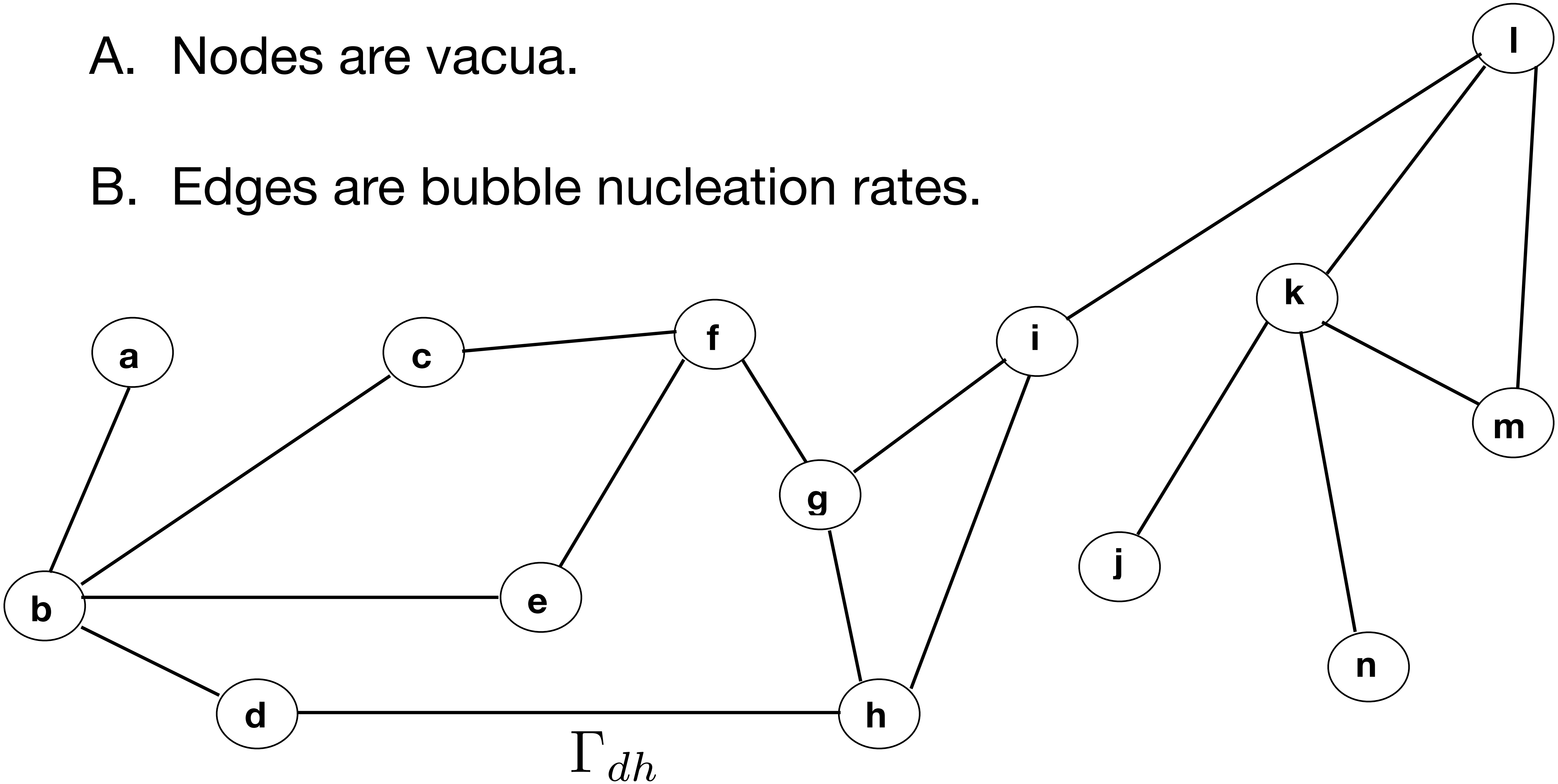


Bubble nucleation and graphs

The set of vacua **i**, and the nucleation rates, define a weighted graph:

A. Nodes are vacua.

B. Edges are bubble nucleation rates.



Network is input for a cosmological bubble nucleation model.

A Simple Model of Bubble Nucleation

- Introduced networks, now we can ask what the network structure predicts for bubble nucleation models.

- Simplified model of bubble nucleation (assume bubbles do not collapse). Based on J. Garriga, D. Schwartz-Perlov, A. Vilenkin, and S. Winitzki,

  see also D. Harlow, S. H. Shenker, D. Stanford, and L. Susskind

N_j : number of bubbles in vacuum j.

Γ_{ij} : bubble nucleation rate from vacuum j to vacuum i.

$$\frac{d\mathbf{N}}{dt} = \Gamma \mathbf{N}$$

A Simple Model of Bubble Nucleation

Bubble nucleation determined by $$\frac{d\mathbf{N}}{dt} = \Gamma \mathbf{N}$$

Solution: $$\mathbf{N} = e^{\Gamma t}\mathbf{N}_0 = \sum_p a_p e^{\gamma_p t}\mathbf{v}_p$$

$\mathbf{N}_0$: initial vacuum numbers

γ_p , $\mathbf{v}_p$: eigenvalues, eigenvectors of Γ

a_p : initial conditions

A Simple Model of Bubble Nucleation

Late time solution:

Let γ_0 , $\mathbf{v}_0$ largest eigenvalue, eigenvector of Γ

As $t \to \infty$ $\mathbf{N}$ is dominated by the largest eigenvector of Γ

$$\mathbf{N} \to a_0 e^{\gamma_0 t} \mathbf{v}_0$$

However, the entries become infinite as $t \to \infty$

Define the fractional distribution of vacua: $\mathbf{p} = \mathbf{N}/|\mathbf{N}|$

$\mathbf{p}$ is well-defined, and independent of initial condition.

A non-trivial distribution in p indicates vacuum selection!

To solve we need to determine Γ in our model, and ensure that the answer makes sense

(i.e. no negative entries in **p**).

Steps for vacuum selection in string theory:

1. Construct the network of vacua in the string landscape.
2. Model cosmological evolution using $\Gamma_{ij} \to$ differential equation.
3. Solve for the largest eigenvector of Γ , which provides a notion of vacuum selection.

First step: construct the nodes (vacua) and edges (nucleation rates).

Geometric networks in string theory

What data defines a metastable vacuum in string theory?

1. Choice of compact manifold, perhaps with special holonomy.

2. Choices of objects: flux and branes.

3. Choice of solution to equations of motion.

These are what I would call “typical string vacua”, at least in the corners of the landscape that we understand best.

These gives the **nodes** in the network.

In general these vacua are hard to construct explicitly, so we will consider a coarse-grained toy model: **nodes = geometries**.

Bubble nucleation between vacua

Transitions between vacua in the landscape seem rather complicated. One generically expects tunneling between many, if not all, of the vacua, and the tunneling is outside of the realm of effective field theory.

However, there is a set of transitions, **geometric transitions**, that might play a special role.

These geometric transitions give the geometries a **graph structure** (see talk of Taylor and Wang).

These are completely general in string theory, but to discuss them I will specialize to F-theory.

F-theory

- IIb with 7-branes, varying axiodilaton.
- More general than IIb with D7-branes. Allows for strong coupling.
- Mathematically described by a Calabi-Yau elliptic fibration over base B, where B is the internal space.

$$y^2 = x^3 + fx + g$$

- Singularities of elliptic fibration specify location and type of 7-brane in B.

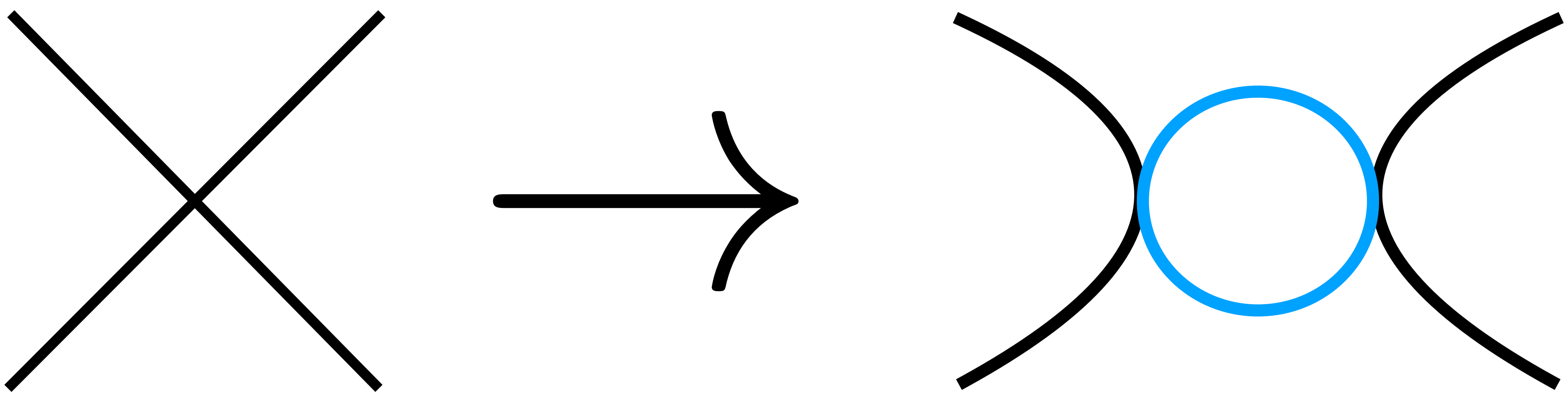
$$\Delta = 4f^3 + 27g^2 = 0$$

- I will mostly phrase things in terms of branes on B, except for technical asides.

Geometric Transitions

- When 7-branes stack up enough and intersect, a new branch of moduli space appears.

- Blowing up along this cycle separates the 7-branes, and changes the topology of the compact directions in space.



- Technically, $MOV_C(f,g) \geq (4,6)$ or $MOV_P(f,g) \geq (8,12)$

Hayakawa, Wang, Morrison

- This corresponds to a **crepant** resolution of the corresponding fourfold, and is therefore motion in Calabi-Yau moduli space.

Modeling Γ

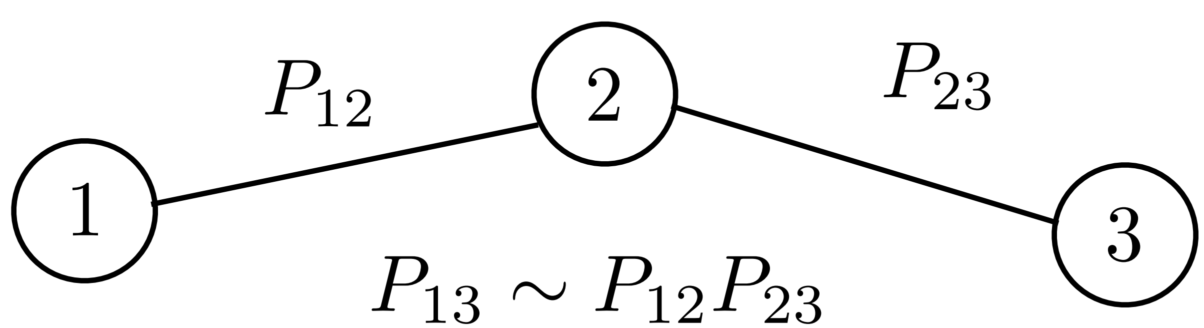
- To calculate Γ we need detailed information about the vacua.
- In general these vacua are not even in the same effective 4d field theory, so analysis goes beyond the original Coleman-De Lucia story.
- **Main assumption: the dominant bubble nucleation process is between geometries that are directly connected by a single topological transition.**

Motivation for nearby geometries

- We want to generalize the Coleman-De Lucia result, which involves a measure of distance between vacua.

- The network structure provides such a distance, somewhat natural as the network captures motion in moduli space.

- Multiple topological transitions require tuning to higher codimension in moduli space, so we expect nearby geometric transitions to be preferential.



- (Before moduli stabilization/quantum effects, these transitions correspond to motion in moduli space. Finite temperature (for instance) could allow for such fluctuations to occur dynamically).

Modeling Γ

Without further information about the vacua (fluxes, vacuum energies, etc.), we consider a simplified model.

$$\Gamma = \alpha \mathbf{A}$$

α constant that determines overall transition rate

$\mathbf{A}$ is the adjacency matrix of the graph: has entry 1 if two geometries are connected, zero otherwise.

In this case, p is the largest eigenvector of the adjacency matrix of the network, also called the eigenvector centrality of the network.

Perron-Frobenius theorem: **p** is strictly positive if **A** is the adjacency matrix of a connected graph, so the interpretation as a fractional vacuum distribution is sensible!

Concrete networks of geometries

Networks of geometries

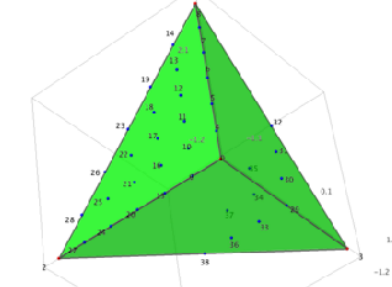
- Both are ensembles of Calabi-Yau's associated with toric varieties. The one we are interested in admit a combinatorial description as triangulations of polytopes.

  1. The Tree ensemble: Calabi-Yau Elliptic fibrations over toric 3-folds.

  2. The hypersurface ensemble: Calabi-Yau hypersurfaces in toric 4-folds. Batyrev
     Kreuzer, Skarke

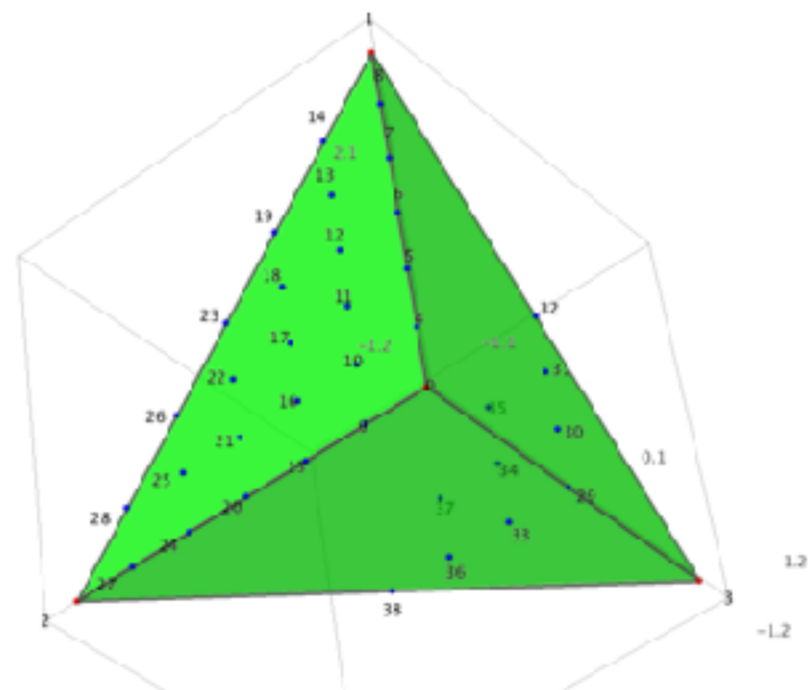
- In both cases, nodes represent Calabi-Yau geometries, and edges represent blowups between the geometries.

The Tree Ensemble



Halverson, CL, Sung

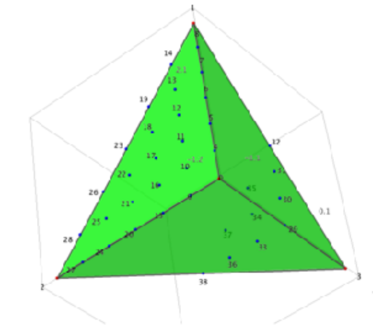
1. Start with a weak Fano toric 3-fold base, corresponding to a triangulated 3d reflexive polytope. Defines a **Fan** with rays v_i .



2. Blowup subvarieties to reach a new toric base. Combinatorially described by adding new ray v_e to the fan, corresponding to a new exceptional divisor D_e .

$$v_e = \sum_i a_i v_i$$

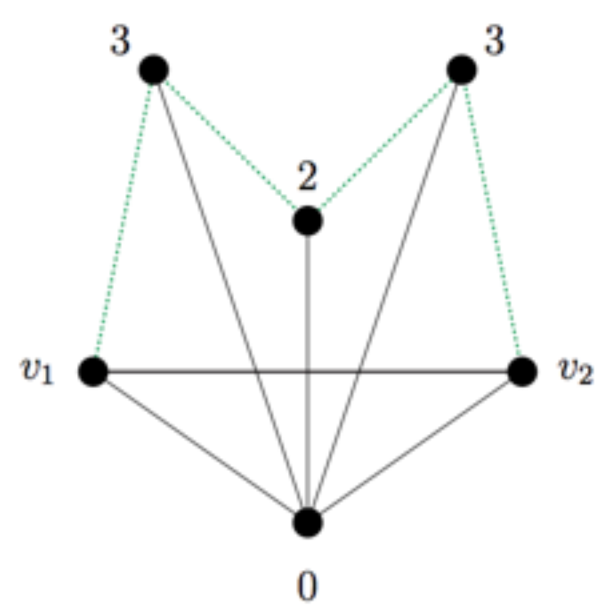
The Tree Ensemble



$$v_e = \sum_i a_i v_i$$

- Define the height of a blowup as $h = \sum_i a_i$
- In general, can blow up along

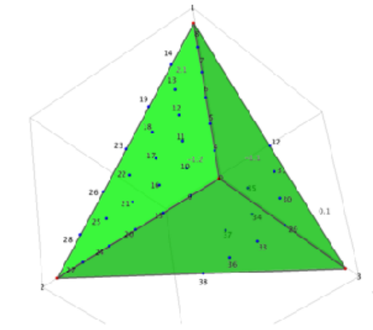
1. Toric curves <—> edges in the triangulated polytope.



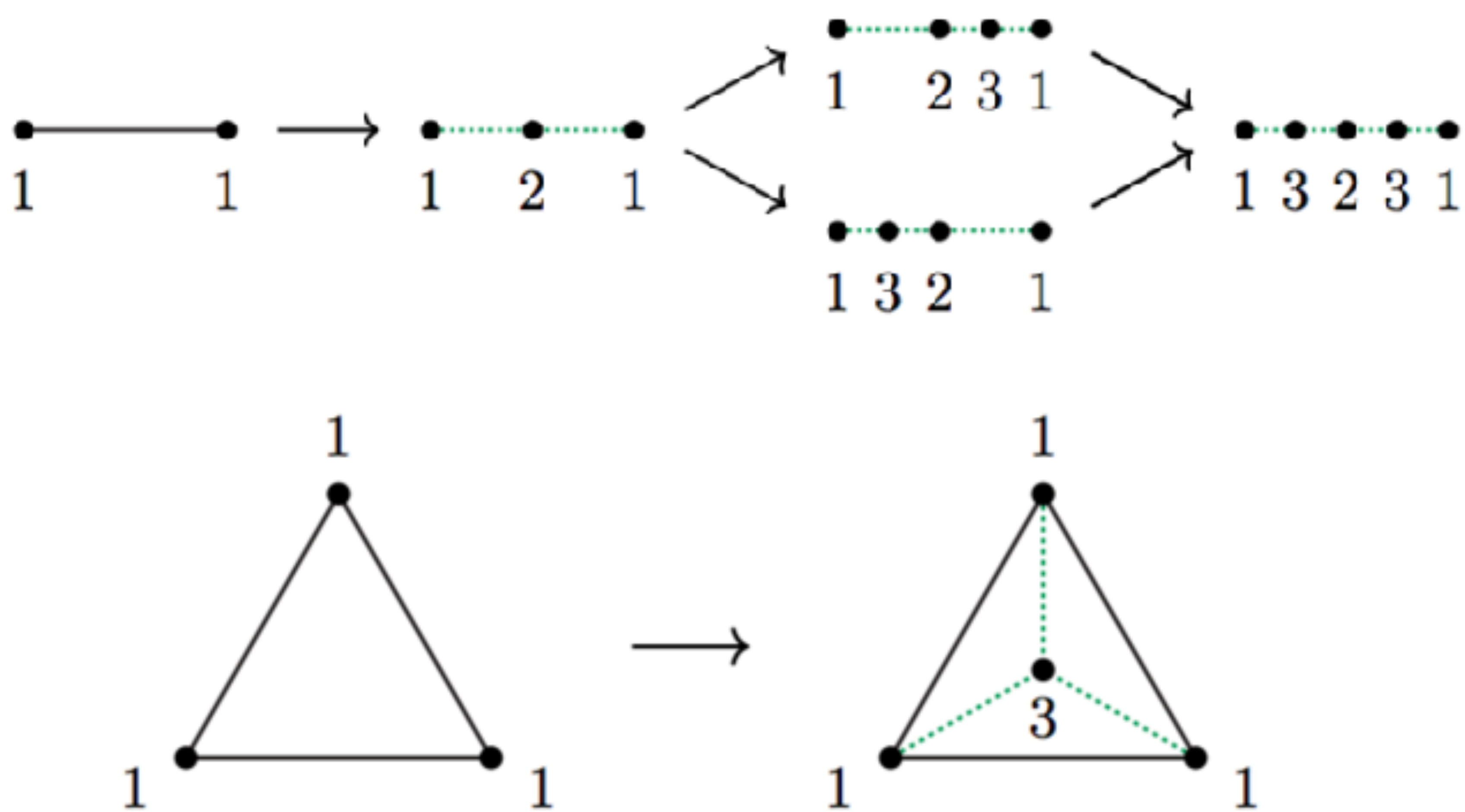
Growing a tree above the edge!
Disclaimer: not a graph theory tree.

2. Toric points <—> faces (triangles) in the triangulated polytope.

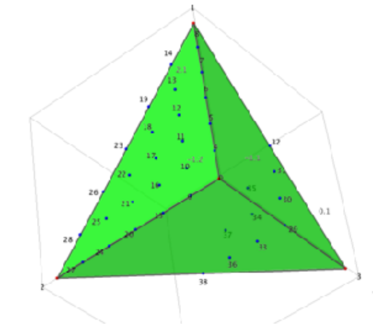
The Tree Ensemble



- To visualize, it's easier to project all rays back onto the polytope, so 'growing a tree' corresponds to subdividing edges and faces.



The Tree Ensemble



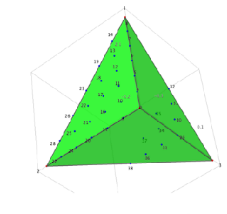
- Calabi-Yau elliptic fibrations over these bases form a connected moduli space, related by topological transitions, **if** a technical condition is satisfied, which is

$$MOV_{D_e}(g) < 6 \text{ or } MOV_{D_e}(f) < 4$$ Hayakawa, Wang

- A sufficient condition to ensure that each Calabi-Yau is connected in moduli space limits the possible blowups in a given local patch to a finite set, rendering the ensemble finite.

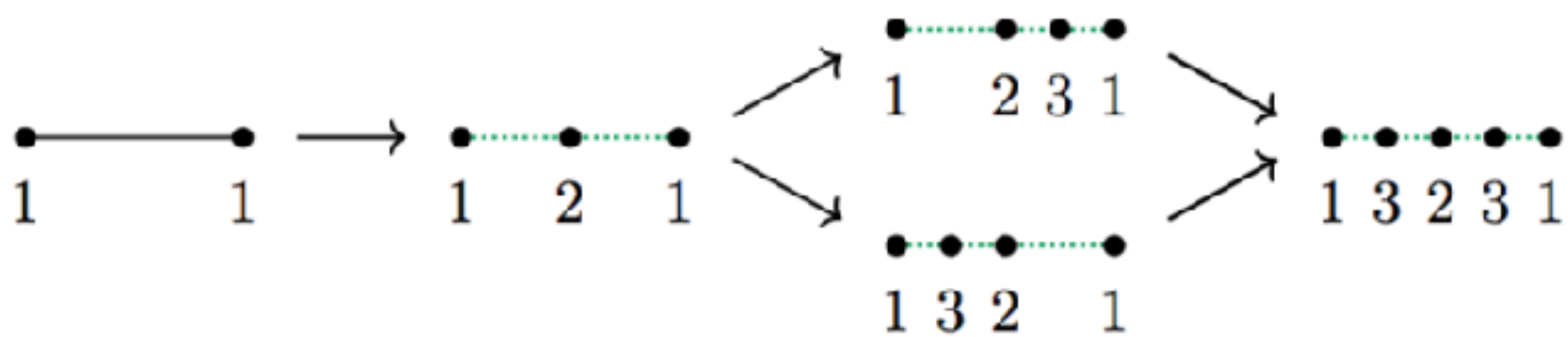
$$MOV_{D_e}(g) < 6 \leftrightarrow h(v_e) \leq 6 \text{ for all } v_e$$ Halverson, CL, Sung

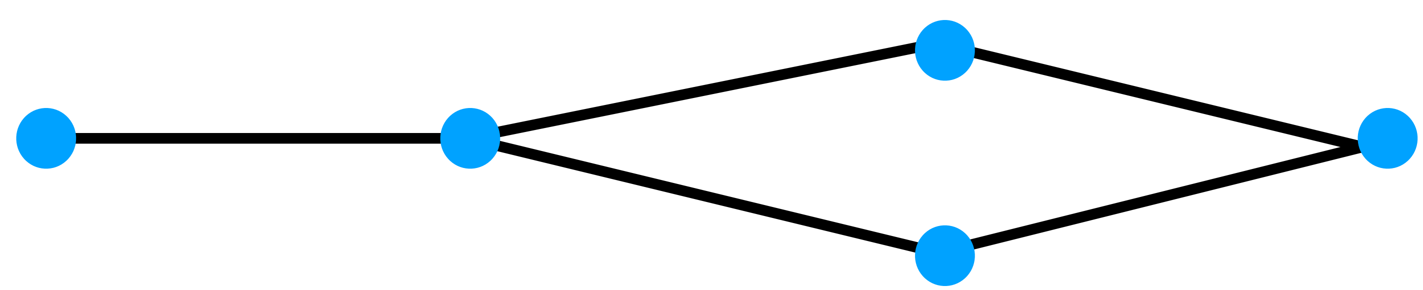
- The topological transitions give this ensemble a network structure: **geometries are nodes**, and **topological transitions are edges**.



The Edge Network N_E

- First consider blowup of curves. Toric curves correspond to edges in the triangulation.

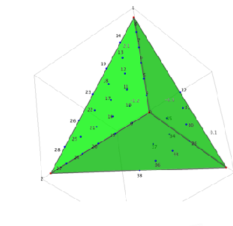




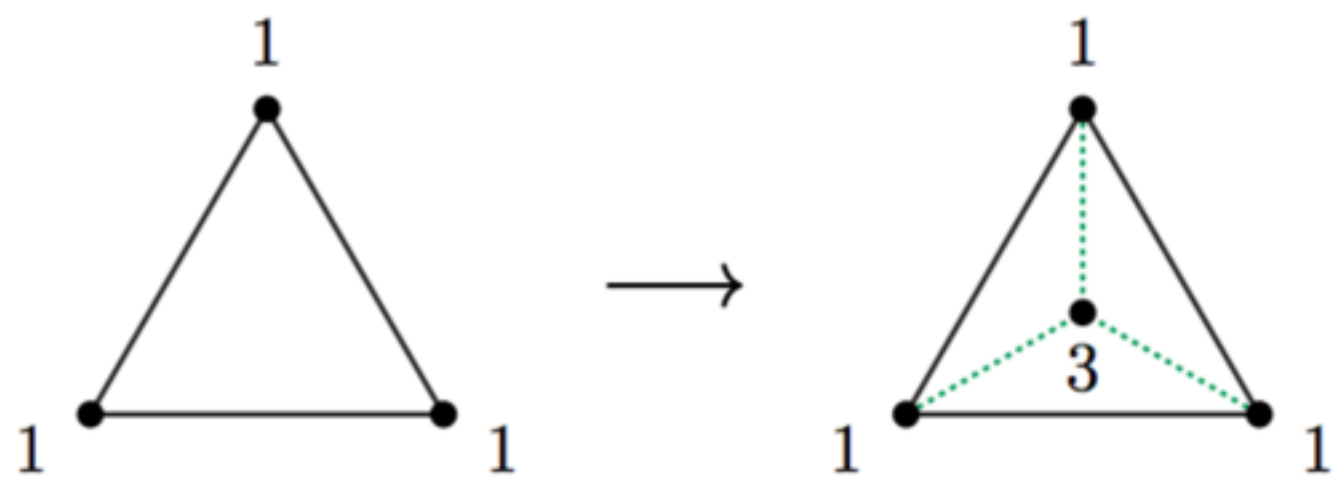
- A single toric curve, corresponding to an edge in the triangulation, admit 82 configurations of blowups.

  These configurations form a network N_E , with **82** nodes and **1386** edges.

The Face Network N_F

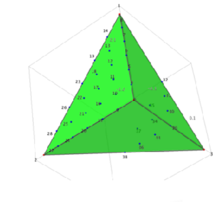


A toric point corresponds to a triangle in the triangulation.



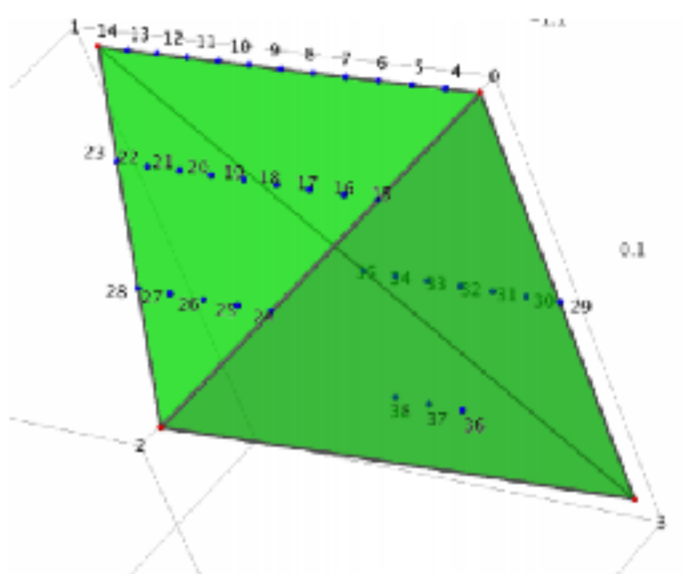
These configurations form a network N_F with **41,873,645** nodes and **100,036,155** edges.

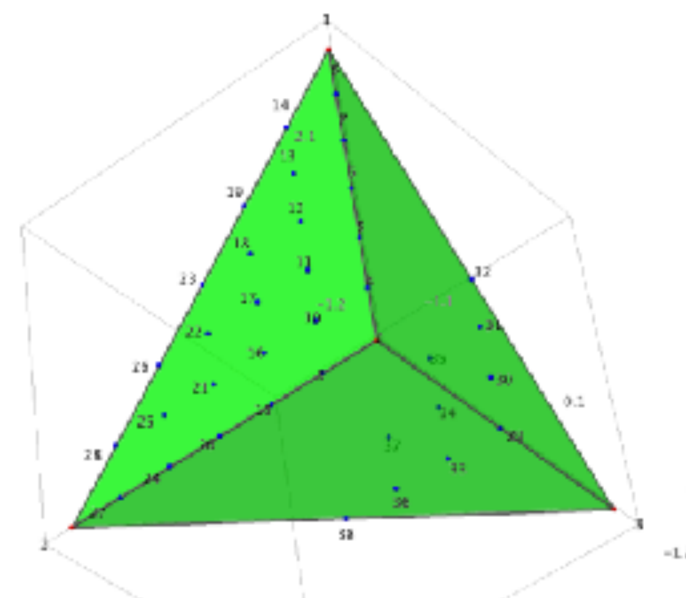
The Tree Network



Ensemble of tree geometries overwhelmingly generated by blowups of toric varieties corresponding to two reflexive polytopes with the most edges and triangles.

Each has 108 toric curves (edges) and 72 toric points (triangles) when triangulated.

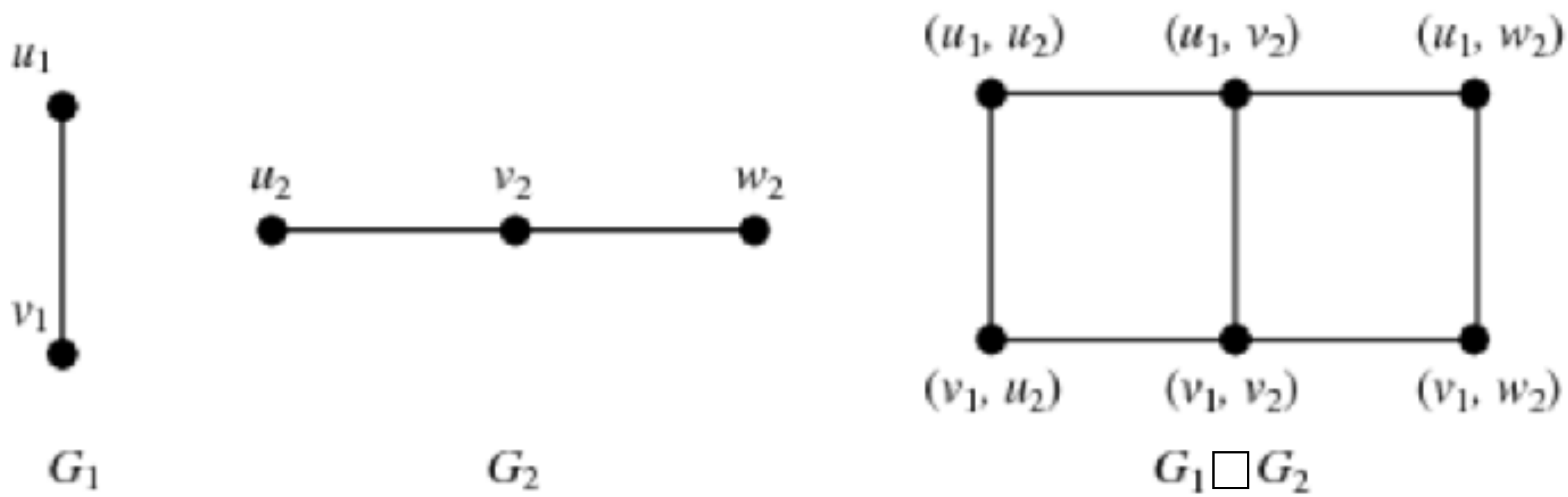




$$|S_{\Delta_1^\circ}| = \frac{2.96}{3} \times 10^{755} \qquad |S_{\Delta_2^\circ}| = 2.96 \times 10^{755}$$

The Tree Network

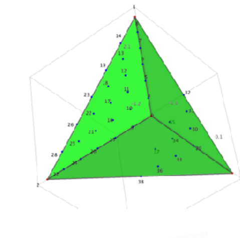
- A generic network with 10^{755} nodes would be completely intractable, but this network factorizes into a cartesian product of graphs:



Wolfram

Cartesian product = $\square$

The Tree Network

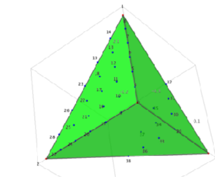


- The tree network N_{tree} factorizes as

$$N_{\text{tree}} = N_E^{\square\,108} \,\square\, N_F^{\square\,72}$$

- Simply put, two geometries in the Cartesian product are adjacent if they are related by a single blowup in a single local patch.

By understanding N_E and N_F we can learn about N_{tree} !

Some properties of the geometries 

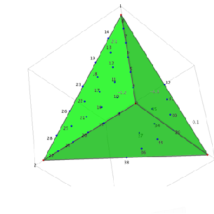
- Toric varieties corresponding to reflexive polytopes admit smooth elliptic fibrations.

- Blowups on any face force non-Higgsable clusters on all divisors corresponding to points interior to that face.

- Fraction of geometries that have non-Higgsable clusters:

$$1 - 1.01 \times 10^{-755}$$

- Further blowups force higher order non-Higgsable clusters.

Some properties of the geometries 

- Enormous number of geometries, too many to scan.

- However, understanding the construction algorithm allows us to read off the minimal geometric gauge group in terms of simple combinatorial data, with probability ≥ 0.999995

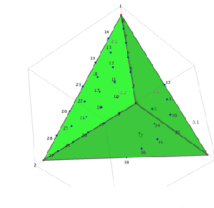
$$G \geq E_8^{10} \times F_4^{18} \times U^9 \times F_4^{H_2} \times G_2^{H_3} \times A_1^{H_4} \qquad U \in \{G_2, F_4, E_6\}$$

$$rk(G) \geq 160 + 4H_2 + 2H_3 + H_4$$

H_2, H_3, H_4 are number of height 2,3,4 blowups.

There actually are, definitively, over 10^500 string geometries.
and we have an exact lower bound.

We can actually control this ensemble, through precise knowledge of the construction algorithm.

Some properties of the geometries 

- Non-Higgsable clusters, so generic points in moduli space are strongly coupled. Do any of these geometries admit a Sen limit?

- In recent work, we worked out requirements for the existence of a Sen limit in the following cases: Halverson, CL, Sung

  1. Toric bases.

  2. Algebraic bases constructed from gluing local patches, where the local patches are crepant resolutions of orbifold singularities.

- Applied to the tree ensemble, the fraction that admits a Sen limit is

$$3 \times 10^{-391}$$

Not only do generic points in moduli space have strong coupling points, but all subloci do as well!

The Hypersurface Network Δ°

- Nodes are Calabi-Yau threefold hypersurfaces in toric fourfolds, connected by topological transitions.

- The topological transitions we consider are the ones inherited from transitions in the ambient toric fourfolds, which correspond to 4d reflexive polytopes.

- Transitions encoded by adding points to or removing points from 4d reflexive polytopes.

- We can therefore consider the nodes to be polytopes.

The Hypersurface Network Δ°

- Two polytopes Δ_1° and Δ_2° are connected by an edge in the hypersurface network if one can get Δ_1° from deleting one or more vertices from Δ_2°, along with an GL(4,Z) transformation.
(without passing through an intermediate polytope).

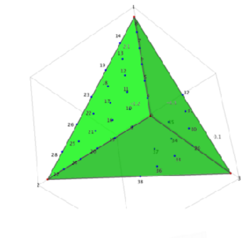
- There are 473,800,776 4d reflexive polytopes, too many to handle right now.

- Restrict to polytopes with 10 or less vertices.

- Network was **11,631,590** vertices, and **43,547,394** edges.

Examples of geometry selection

The Tree Network

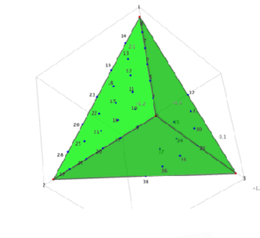


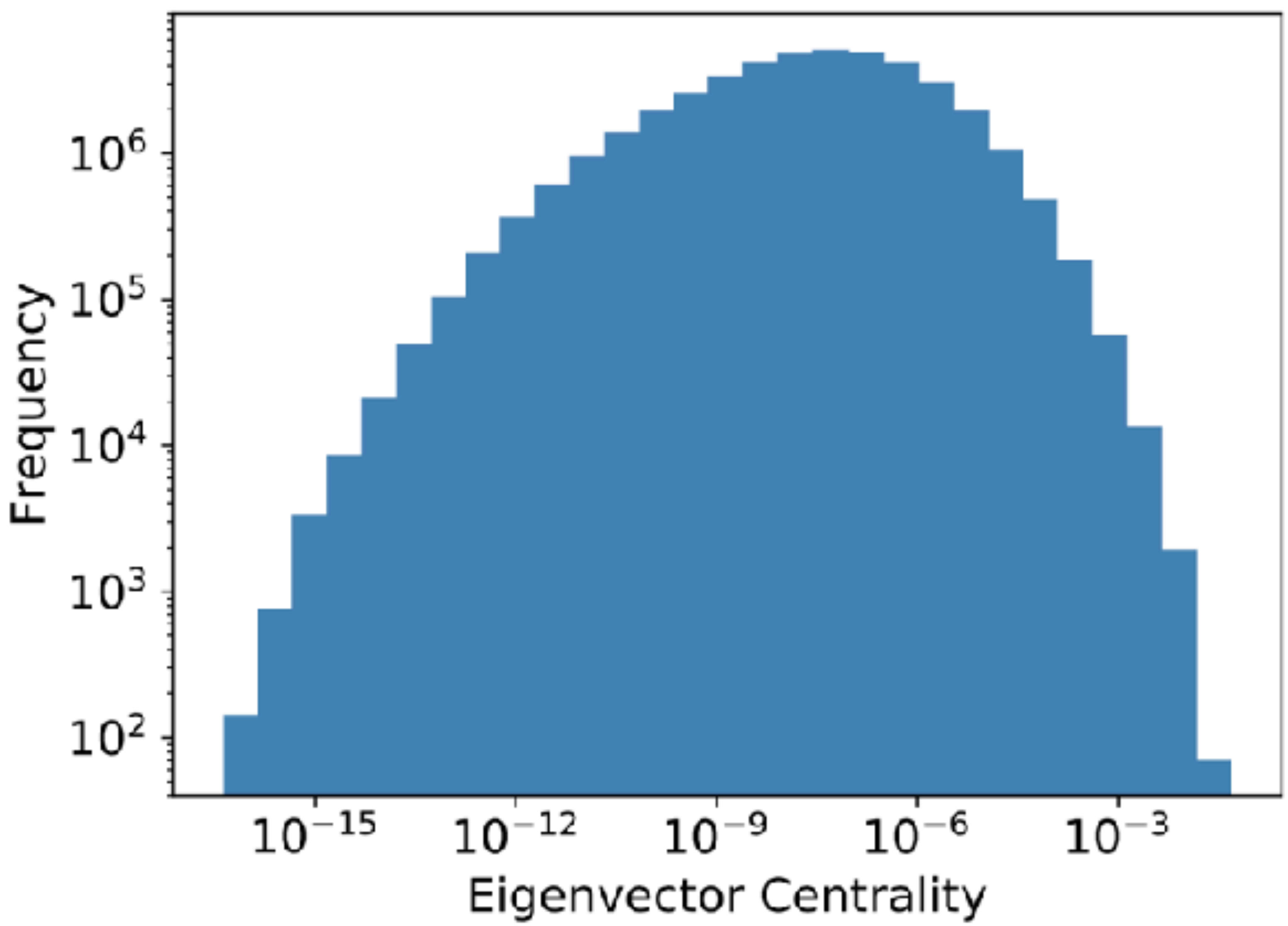
Recall $N_{\text{tree}} = N_E^{\square\,108} \,\square\, N_F^{\square\,72}$

A useful fact is that $\mathbf{p}(N_{\text{tree}}) = \mathbf{p}(N_E)^{\otimes 108} \otimes \mathbf{p}(N_F)^{\otimes 72}$

We therefore need to calculate $\mathbf{p}(N_E)$, $\mathbf{p}(N_F)$

N_f

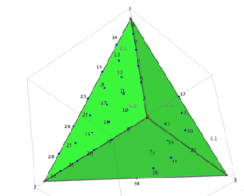




$\mathbf{p}(N_F)$

Largest entry is 0.07, 98 percent of the entries are at least a factor of 1000 smaller. Ratio of largest to smallest is ~ 10^7

The Tree Network



N_E a much smaller, less interesting network, but still a non-flat distribution.

Full tree network: Ratio of largest to smallest eigenvector centrality ~

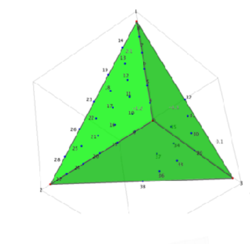
$$7 \times 10^{1457}$$

This is a measure of maximal geometry selection in the tree network.

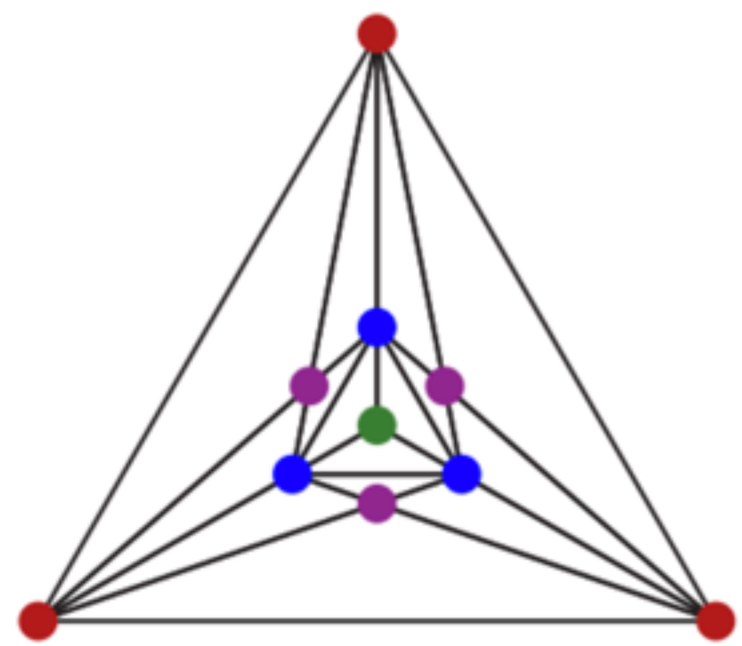
While it is a toy model, it is a coarse grained toy model of actual huge string networks!

Main lesson: non-trivial graph structure in networks of F-theory geometries gives rise to geometry selection in a simple bubble nucleation cosmology.

Physics of the selected node



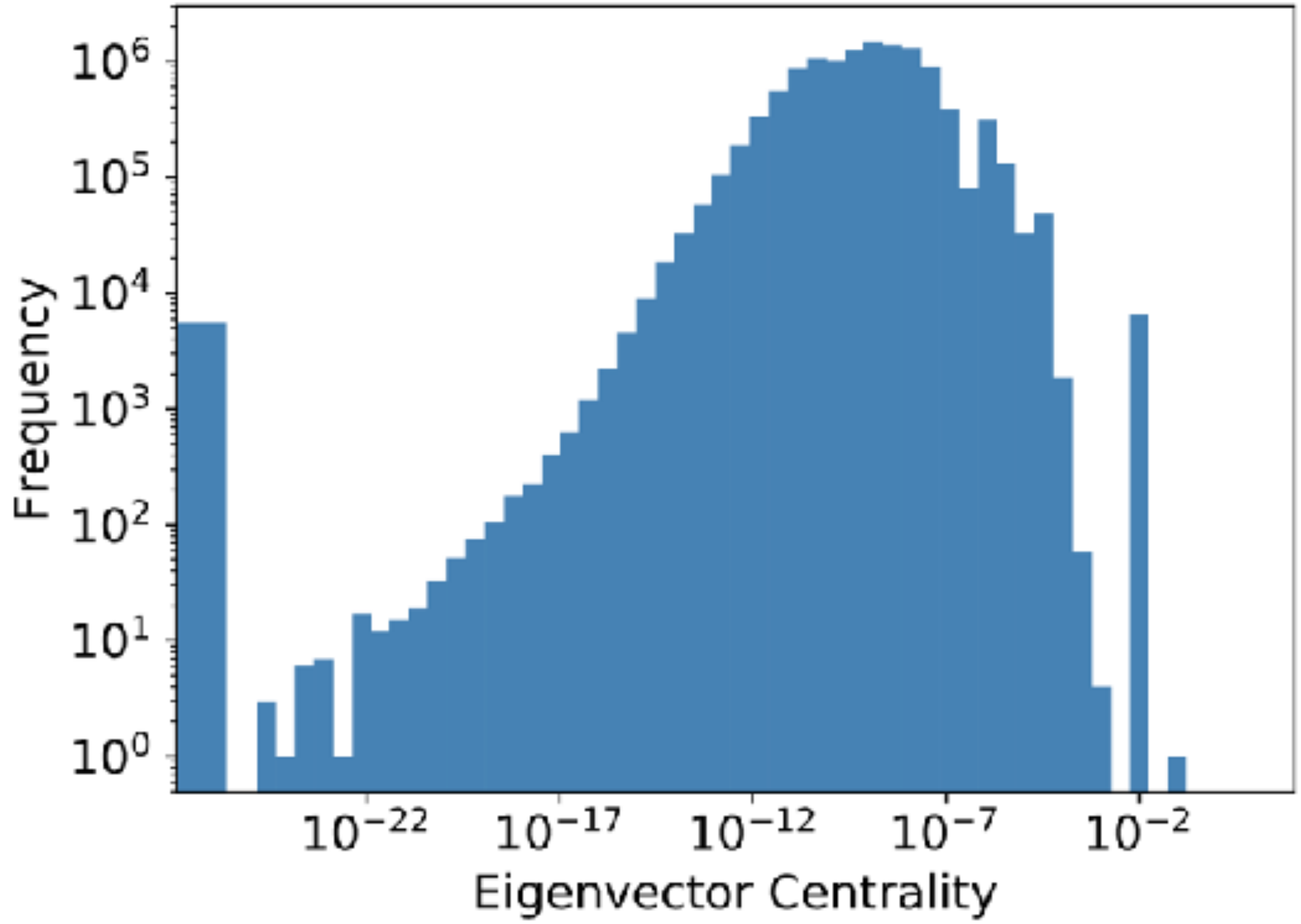
Selected node in N_F :



$G_2 \times (SU(3))^3$

Full tree network: $E_8^{37} \times F_4^{85} \times G_2^{220} \times SU(2)^{320}$

The Hypersurface Network



Ratio of largest eigenvector centrality to smallest is 10^{24}

Peak on the right is from polytope connected to most connected node, which also has the largest eigenvector centrality.

Peak on left is due to cutoff at 10 vertices.

Recap

- The landscape of string vacua naturally has a network associated with it: nodes = vacua, edges = bubble nucleation rates.
- Introduced two large networks of string geometries.
- We considered a toy model for vacuum selection using network science that can give rise to large selection factors, and we demonstrated that is does in concrete geometric networks.

Musings

- Ways to to move away from the toy regime: fluxes, mobile branes, vacuum energies.

- Transitioning between vacua in string theory interpolates between different effective field theories, important to understand better.

- Dynamics of geometric transitions and relevant instantons need to be calculated.

- Would be interesting to apply to Taylor-Wang graphs.

Thanks!